1991

Adaptive dynamic control for robotic manipulators

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ADAPTIVE DYNAMIC CONTROL FOR ROBOTIC MANIPULATORS

A thesis submitted in fulfillment of the requirement
for the award of the degree of

DOCTOR OF PHILOSOPHY

from

THE UNIVERSITY OF WOLLONGONG

by

Ming LIU (M.E.)

Department of Electrical and
Computer Engineering

(February 1991)

ACKNOWLEDGMENTS

Firstly, I am indebted to my supervisor Professor Christopher D. Cook, Head of the Department of Electrical and Computer Engineering, Chairman of the Automation and Engineering Applications Centre, The University of Wollongong, for the supervision, help and excellent conditions he gave me to accomplish my research and complete this dissertation. I wish to give my thanks to my other supervisor Professor Brain H. Smith, former dean of the Faculty of Engineering, The University of Wollongong, for his every help during my study at this university.

I would particularly like to thank Associate Professor Tony S. Ng, Senior Lecturer Dr Moneeb A. Magdy, and Senior Lecturer Dr Geoffrey W. Trott, Department of Electrical and Computer Engineering, The University of Wollongong, for the inspiring discussions we had together and the assistance they gave me. I am also grateful to Bronwyn J. Evans for reading the drafts of this thesis with great care and undertaking some simulations for it.

I want to express my heartfelt gratitude to all staff members of the Automation and Engineering Applications Centre at The University of Wollongong for providing a very warm environment, computing facilities, robot equipment and financial assistance for me in my research and study.

My thanks are also given to all staff members in the Department of Electrical and Computer Engineering, The University of Wollongong for their help during the time I worked on this thesis.

ABSTRACT

The investigations and research introduced in this dissertation are about the development of novel adaptive control strategies for trajectory tracking motion control of robotic manipulators.

The overall control system structure is based on a two-component control law: one is a full model inverse dynamic control law (the computed torque control scheme) which is a non-adaptive control component and the other is an adaptive control component. Because the dynamic parameters used in the first control law do not usually match the real unknown parameters of the controlled robot dynamics, the resultant tracking error equation is not exact. For this tracking error dynamics a decentralized system structure is obtained by decomposing the signal stimulating the error system into two parts: one is the paramaterized dominant input and the other takes into account the interconnections between each subsystem. Utilizing this decentralized system structure, the adaptive control component is designed for each subsystem. The main results are two novel adaptive algorithms for robot trajectory following control based on the decentralized system structure.

The adaptive control components are designed as compensators for the paramaterized dominant input for each subsystem. The first algorithm gives quantitative results on the boundedness of both position tracking errors and parameter estimation errors and their convergence rates under the assumptions that the interconnections are bounded by some constants. Further investigations showed that the boundedness of the interconnections are related to the boundedness of the overall system states (positions and velocities). As a result of this the second algorithm is developed. This algorithm, though more complex, gives complete theoretical results on the convergence of state tracking errors (position

errors and velocity errors) and parameter estimation errors and their convergence rates without requiring the assumptions used in the first algorithm. The bounded magnitudes of adaptive control torque are ensured by the method as well.

Both methods have the benefits that the accelerations of the robot joints need not be measured explicitly, i.e., only system states (position and velocity signals) are needed in the feedback and no acceleration sensors are required; the inverse of the estimated inertial matrix is always non-singular, and only the diagonal elements of this inverse need be computed.

CONTENTS

NOTATIONS AND SYMBOLS

$n \in R_+$:	number of joints of robot arm;
$q=q(t) \in R^n$:	generalized joint position vector of robot arm with n joint;
$\dot{q}, \ddot{q} \in R^n$:	velocity and acceleration vectors of robot arm;
$q_d=q_d(t) \in R^n$:	generalized reference position vector;
$\dot{q}_d, \ddot{q}_d \in R^n$:	reference velocity and acceleration vectors;
$e=q_d-q \in R^n$:	position tracking error;
$\dot{e}, \ddot{e} \in R^n$:	velocity and acceleration error;
$D(q)=D^T(q) \in R^{nxn}$:	positive definite inertial matrix;
$\hat{D}(q)$:	an estimate of $D(q)$;
$\hat{D}^{-1}(q)=[\hat{D}(q)]^{-1}$:	inverse matrix of $\hat{D}(q)$;
$h(q,\dot{q}) \in R^n$:	centrifugal, Coriolis forces vector;
$\hat{h}(q,\dot{q})$:	an estimate of $h(q,\dot{q})$;
$g(q) \in R^n$:	gravitational force vector;
$\hat{g}(q)$:	an estimate of $g(q)$;
$u=u(t) \in R^n$:	generalized input torque vector;
$\tau=\tau(t) \in R^n$:	generalized input torque vector;
θ:	system parameter constant vector;
$\hat{\theta}$:	a priori estimate of θ, which is a constant vector;
$\bar{\theta}$:	a priori estimation error vector;
$\hat{\theta}(t)$:	an estimate of a priori estimation error $\bar{\theta}$;
$\omega(q,\dot{q},\ddot{q})$:	measure vector consisting of nonlinear function of $q,\dot{q},\ddot{q}$;
$\delta(q,\dot{q})$:	measure vector consisting of nonlinear function of $q,\dot{q}$;
lower case letter, say, $m \in R^n$:	a vector defined on the n-dimension real vector field;

m_i:	the i-th element of vector m;
upper case letter, say, $M \in R^{nxl}$:	nxl real matrix;
$\Psi(q,\dot{q}) \in R^1$:	kinetic energy function of robot system;
$\Pi(q) \in R^1$:	potential energy function of robot system;
$\{M_{ij}\} \in R^{nxl}$:	another expression of matrix M;
M_i:	the i-th row of M;
M_{ij}:	the i-j-th element of M;
M^T:	transpose of M;
M^{-1}:	inverse of non-singular matrix M;
$(\dot{})$:	derivative with respect to time t;
$\lambda_i(M)$:	the i-th eigenvalue of $M \in R^{nxn}$;
$\max\lambda(M)$ ($\min\lambda(M)$):	maximum (minimum) eigenvalue of real symmetrical matrix M;
s:	differential operator: s(.)=d/dt(.);
$\lvert . \rvert$:	absolute value of scalar (.);
$\lVert x \rVert$:	Euclidian norm of vector $x \in R^n$, defined by $\lVert x \rVert = (\sum_{i=1}^{n} \lvert x_i \rvert^2)^{1/2}$;
$diag\{m_{ii}\} \in R^{nxn}$:	diagonal matrix with m_{ii}, $i \in n$, as its diagonal elements;
$\lVert M \rVert$:	Euclidian norm of matrix $M \in R^{nxn}$, defined by $\lVert M \rVert = (\max_i \lambda_i(M^T M))^{1/2}$;
$v \in R^1$:	a Lyapunov function for overall system;
$v_i \in R^1$:	a Lyapunov function for subsystem i;
L(s):	Hurwitz polynomial of s;
$L^{-1}(s)$:	inverse polynomial of L(s);
$P_L(q)=L(s)\theta(t)L^{-1}(s)$:	a linear operator specified by L(s) and time function $\theta(t)$;
L_i:	length (m) of the i-th link of a given robot;
m_i:	mass (kg) of the i-th link of a given robot;

m_L: mass (kg) of the payload attached on a robot's free end;

A: system matrix in state space equation;

b: input matrix in state space equation;

h: output matrix in state space equation;

Chapter 1

INTRODUCTION

1-1. PROBLEM STATEMENT

One of the most important characteristics of a robotic manipulator is the controllable mobility of its arm. This is achieved by applying appropriate torques to different joints of a robot arm to cause the desired movement of the robot "hand". In doing so, two well defined mappings should be set up. The first mapping, defined by inverse kinematics, specifies, for a given motion of a robot hand in the Cartesian space, the positions and velocities of every joint of the robot's arm at each instant. The second mapping, which is defined from torque coordinate space to the robot's joint coordinate space, reveals the relationship between the torque applied to each joint of the robot arm and the motion of the joint caused by the torque. To define and set up this mapping is the study of dynamics. The mathematical formulation of this relationship is the dynamic equation or motion equation of a robot system.

This thesis focuses on the topic of robot dynamic control. It is about the study of control philosophies based on the available knowledge of a robot's motion equations to produce appropriate torques to give the required motion of the robot joints. Since only the problem of dynamics is concentrated on in this thesis, it is assumed that the inverse kinematic mapping from the Cartesian space to joint space has been accomplished.

The robots' dynamic equations belong to a class of multi-variable non-linear systems consisting of strong couplings between different joints. From the control theory point of view these characteristics are quite difficult to handle. Other control problems are caused by variable payloads when robots are undertaking tasks such as pick-and-place and assembly. In these cases, it is difficult to guarantee consistent dynamic performance as variable payloads will change the system dynamic behaviour.

On the other hand, as discussed later, most contemporary robots controlled by simple non-model-based joint independent control methods work quite well in point to point control with fixed or comparably small variation in payloads. In this sort of application, many modern robots provide acceptable repeatability and position accuracies. However, in trajectory following applications such controllers often cannot provide adequate accuracy. Deviations from the desired trajectory vary depending on the features of the reference trajectories and the arm location within the working envelopes. It becomes even more difficult for industrial robots to maintain good performance when their payloads are widely changing (here "widely changing" means that the change of the payloads is so big that it cannot be ignored compared with the mass of the robot bodies). To overcome such problems, large robots are commonly used to do small jobs (for instance, robots weighing hundreds of kilograms are used to load IC chips onto printed circuit boards on modern electronic assembly lines).

In order to improve performance, model-based control approaches have been developed. Since the overall dynamics of controlled robots are taken into account in controller design, this sort of control method gives improved performance in both position control and trajectory tracking.

However, the application of model-based approaches is limited in most circumstances by two factors.

1). In most cases, it is almost impossible to find the precise dynamic models for a given robot;

2). These methods require complicated on-line calculation at very high speed so that powerful computers are needed in their implementation.

The second problem is alleviated as more powerful micro-computers and processors become available in the market at lower prices. However the first problem is not only a problem in robot control, it is a problem commonly existing in control system engineering applications. It is caused by either extremely complex system dynamics or by a limited understanding of system behavior. In order to control systems with imprecise models, novel control mechanisms with new philosophies such as dynamic system identification, adaptive control, robust control, variable structure systems, self-learning systems, expert systems and intelligent control systems have been exploited. In order to improve the performance of robot motion control, some specific methods have been proposed for particular robot control and some have been introduced into robotics from general control theory research.

This dissertation focuses on using adaptive control methods in industrial robot motion control.

1-2. SURVEY OF PREVIOUS RESEARCH

Robot dynamic control approaches can be classified in several categories:

---- Proportional -integral-differential (PID) control;

---- Computed torque method (CTM);

---- Variable structure system (VSS);

---- Robust control;

---- Adaptive control.

Most commercial robots use classical *PID joint independent control* methods in their controllers. This is because the algorithms are simple and need only small computation power. For some applications they give satisfactory results. Arimoto and Miyazaki [2] showed using Lyapunov's direct method and LaSalle's invariant principle [31] the stability and robustness of PID approach in position control. A key point in their stability analysis is the utilization of the passivity feature of robot dynamics. The limitation of the PID approach is that it only gives good performance in reaching a fixed position rather than tracking a moving trajectory. In these cases the reference trajectories, which are functions of time, introduce time-varying characteristics into the closed loop systems so that the LaSalle's invariant set principle is violated. This is to be expected because the PID control schemes do not utilize complete information of the system dynamics and only local position and velocity feedbacks are used by their system structures.

Computed torque control, proposed by [9], [42], and [66], is a fully model based approach. In this approach, the control torque is computed from the inverse dynamics of the controlled robot. Based on the complete knowledge of all the system parameters and dynamic structure, this approach cancels all nonlinearities and results in a decoupled, autonomous, linear error equation. By proper choice of the position and velocity feedback gains, the desired transient response can be obtained. Theoretically, computed torque control is a perfect approach in handling robot nonlinearities, but practically it suffers from the fact that the precise dynamic models can be obtained only on rare occasions because of the complicated dynamic behaviour of robots. In practice only an approximation of the controlled robot dynamics is available. In contrast to the ideal cases, the system nonlinearities cannot be cancelled completely and the resultant error equation is not exact. The tracking performance of the robot will depend on the modelling error.

The *variable structure system* (VSS) (see, e.g., [63]) is designed in such a way that all system state trajectories in the state space are directed toward some switching planes. Once the system state reaches the switching planes, it slides along them and the system response depends thereafter only on the gradients of the switching planes and remains insensitive to a class of disturbances and parameter variations. This approach is able to control both linear and nonlinear systems. It was introduced into robot control by Sastry and Slotine [52a], Young [68] and Yeung and Chen [67] et al. This switching technique tolerates inaccuracies in the robot models. Some simulation results show that even for relatively small uncertainties in the robot model the sliding model control can achieve better performance than the computed torque method. However the drawback of this approach is that it might lead to a large amount of chattering associated with excessive control torques causing robot actuators to have undesirable high-frequency oscillations. Slotine [55], and Singh [54] proposed some modifications to this method, combining it with robust controller design, to overcome the problems of "chattering".

Due to the complex nature of industrial robot dynamics, it is inevitable that some uncertainties will be introduced in modelling and control. These uncertainties could be attributed to model parameter errors, disturbance torques from within the system or from the external environment, measurement noises, payload variations and so on. To overcome the influences of such uncertainties *robust control* was used in controlling robots. The mechanism of robustness is the same as the concept of maintaining an abundant relative stability for closed loop systems in classical system design, that is, to ensure large enough phase and gain margins. The robust design could be regarded as "worst case" design. As the uncertainties are unmeasurable one can give a "guess" to their upper bound. Based on this bound, a control law is designed so that as soon as the error state penetrates a defined dead zone concentric with the origin of the state space control, input energy vanishes or drops to ensure the error always remains inside the dead zone rather than escaping from it. Spong et al ([59], [60] and [61]) used this method to improve computed torque control and gave a stability proof. In Spong's design the robot actuator dynamics and measurement noise were taken into account. Using the decentralized control technique Cvetkovic and Vukobratovic [15], and Gavel and Hsia [17] also developed robust control methods by means of local feedback. Ha and Gilbert [20] investigated the problem of robustness of nonlinear system tracking which was an extension of their early research in robust control of industrial robots [21]. A robust control design scheme using the input-output stability theory can be found in [8]. The system performances of these nonlinear methods rely on the accuracy of the robot dynamics model. As a dead zone is employed zero tracking error cannot be guaranteed.

The application of *adaptive control* to robotic manipulators is motivated by the desire to improve dynamic control accuracy in cases where some or all system parameters are unknown. Adaptive control has developed in two parallel branches: self-tuning control for discrete-time systems (see, e.g., [4], [11], and [19]) and model following control for continuous-time systems (see, e.g., [45], [46], [48], [49], [51], [57], [30] and [37]). For a class of linear system which has some (constant or slowly time-varying) unknown model parameters and uncertainties, the adaptive controller monitors the system input, state or output to refine system dynamic related information by means of a parameter estimator or a certain adaptation mechanism. The outputs of this estimator could be estimates of unknown system parameters (indirect control) or the controller up-date parameters (direct control). Using this information, a control action can be applied to the controlled system so that the performance of the closed loop system becomes optimal or acceptably sub-optimal. This feature makes the method adaptable to some systems with unknown or slowly changing parameters.

In self-tuning systems usually a least square estimator or similar is used and the control indexes can be set by minimum-variance criterion, generalized minimum-variance, or pole placements. In most cases stochastic noise is taken into account as well.

In the case of continuous-time adaptive model following control, a reference model with the desired dynamics is first defined. The control objective is to design a feedback/feedforward controller with adaptive mechanisms so that the closed loop system will have almost the same dynamics as that of the given reference model. Normally the design is based on local parameter optimization (MIT law), the Lyapunov direct method, or Popov's hyperstability theory.

Relatively early studies of applying adaptive control to robots were carried out by Dubowsky and DesForges [16], Lee and Chung [32], Balestrino, Maria and Sciavicco [6], Koivo and Sorvari [29], Lim and Eslami [35][36], Singh [54], Nicosia and Tomei [50], Choi, Chung, and Bien [10], Han, Hemami, and Yurkovich [22], and Hsu and Bodson [24] et al. All of this work used the model reference technique rather than the self-tuning method. Since there are still problems in using discrete techniques to describe and analyse robot dynamics because of their nonlinear behavior a few investigations such as [41] were carried out by using self-tuning models.

As adaptive control was originally developed for linear time-invariant systems, its direct application to robots may encounter some problems because of the robot's nonlinear dynamics. Work described in [16], [50], [35] and [39] uses a quasi-linear system with time-varying unknown parameters to describe the robot model and applies the Lyapunov method to design the controller. Since these parameters are functions of the system state (positions and velocities), they will change as the robot arm moves. This causes convergence problems for the controller because the estimator is designed for constant parameters. Unfortunately, the estimation of time-varying parameters is still an open question in adaptive control. Hence for the purposes of using adaptive control an assumption that the system dynamic parameters are slowly changing or even frozen during the adaptation is necessary. Obviously, this assumption counteracts the significance of the schemes themselves.

Recently an important feature of robot dynamics has been given significant emphasis in robot controller design (see [5], [12] and [55]). This feature is the fact that the equations of motion for rigid robots can be written in such a way that the equations appear linear in some system parameters, such as mass, inertia and payloads. In this formula the equation of motion is rearranged as a multiplication of a generalized state matrix (in the case of whole system description) or a generalized state vector (in the case of subsystem description) and a system parameter vector. The former is called the generalized state matrix or vector because its elements are composed of some nonlinear functions of positions and velocities rather than pure positions and velocities which are the real states of the nonlinear system. Indeed, using this formulation all these nonlinear functions must be worked out. However, as these functions depend on the kinematics, they can be obtained by studying the configuration of the robot system. With this formulation, linear adaptive techniques can be used directly since all parameters of interest (which are unknown or partly unknown) are constants.

Another important property which has been widely used in robot controller design is the passivity of the robot dynamics. The physical meaning of passivity is that the robot's dynamics consist of a system which will neither consume nor generate any energy itself. Mathematically, this feature can be expressed as a relation between the input energy supply ratio and output energy ratio. As was mentioned in the beginning of this section, Arimoto [2], used this feature to prove the stability of a PID controller in robot position control. This idea has been extended by Slotine and Li [55][56][57] in their stability analysis of the tracking controller design. A recent survey in this topic can be found in [23].

These two important properties have provided a basis for many developments in robot motion control in the past few years. As a result of this, two methods have been exploited. The first one was the inverse dynamic scheme which was first proposed by Craig. The second one, proposed by Slotine and Li, was based on both properties.

1-3. CONTRIBUTIONS OF THIS DISSERTATION

In the inverse dynamic adaptive control schemes (see, e.g., [12], [13] and [43]), the adaptive controller design is based on the system structure of the computed torque method. Using this system structure and in cases where there exist some parameter estimation errors, this sort of adaptive control scheme suffers from :

1). The inverse of the estimated inertial matrix must be calculated from time to time. This makes the computation much more complicated especially for robots having many degreeds of freedom;

2). Based on this system structure, the inverse of the inertial matrix is calculated using the updated parameter estimates and the positions. It must be ensured that the estimates of the unknown system parameters always lie in a certain known area in the parameter space so that the resultant inertial matrix is always non-singular. This restriction requires a priori knowledge of "reasonable" sizes of the robot system's unknown parameters. Also the estimators must have special functions such as "cut off" effects to prevent up-dating when the estimates achieve the parameter boundaries or projecting effects to map unreasonable estimates into the domain which ensures the resultant inertial matrices are always non-singular.

3). Using this system structure, the adaptive control is based on the tracking error dynamics which is a set of second-order linear systems forced by a stimulating term related to the parameter estimation errors and system states. Since the accelerations are also involved in this term, this adaptive control law requires their measurement as well. This is not desireable as the direct measurement of the accelerations may introduce significant noise.

In this thesis, a novel system architecture giving a controller structure in which the adaptation law appears as an additional term to compensate the tracking errors of the inverse dynamic scheme (computed torque method) is presented. For the error dynamics obtained by using the computed torque method, the whole n dimension system is treated as n scalar subsystems. This leads to a decentralized system architecture with interconnections among the different subsystems. For each subsystem, the dominant term of the estimation error is parameterized and an adaptive control law is designed. The contributions of the work presented in this thesis include two decentralized system adaptive control algorithms.

In the first algorithm, the acceleration measurements are avoided by introducing filter operators. The Lyapunov direct method is used in controller design and the analysis shows that if the interconnections among different subsystems are bounded by some constants then a quantitative boundednesses of the position tracking errors and the parameter estimates can be obtained. The second algorithm, which is an enhancement of the first, has a two-component controller structure which is similar to that of the first. By using the linear operator proposed in [48], the measurement of the accelerations can still be avoided and a different state realization of the error equation is obtained. Further, it has been shown that the interconnections among the error subsystems are bounded by the overall system states (positions and velocities). Based on these properties of the interconnections and with the development of a new lemma, an adaptive control law is derived using the Lyapunov direct method [33][64]. As a result of this, the quantitative boundedness of a residual set, to which the state tracking errors and parameter estimation errors converge, is obtained. In this method the assumption that the interconnections are bounded by constants is removed and the size of the resultant residual set which deponds on the magnitudes of the interconnections, the maximum displacements and velocities of the reference trajectories can be derived.

The merits of the methods can be summarized as follows:

i). Only n elements of the inverse of the estimated inertial matrix need to be computed. This will reduce the computing time significantly, campared with previous methods (see 1) above).

ii). In the new system architecture proposed, the parameter estimates used in the computed torque control component are fixed by a set of a priori estimates and the real time estimates of the system unknown parameters are only used to update the adaptive control component rather than to update the computed torque schemes. This allows the non-singularity of the inertial matrix estimates to be ensured by a proper choice of the a priori estimates in the non-adaptive control component. In this way, restriction 2) given above is removed.

iii). In the schemes proposed, the introduction of a filter operator means that the explicit measurement of the accelerations of the robot's joints can be avoided and no acceleration sensors are needed.

iv). In the second algorithm proposed, quantitative convergence results, which are related to the interconnections, uncertainties and reference trajectories, are obtained.

1-4. OUTLINE OF THIS THESIS

The thesis is organized as follows:

In Chapter 2, the equations of motion of the robot arm's dynamics are introduced in terms of the Lagrangian formulation. To obtain a thorough understanding of the dynamic behaviour, some important properties of the equations are discussed. In dynamic control, two basic motion control problems – position control and trajectory following control – are reviewed. For position control, the robust properties of the PD control scheme is introduced; and for trajectory following control, the computed torque scheme is restudied. Then the necessity of adaptive control is demonstrated. Among various schemes, Craig's adaptive controller [12][13] is introduced.

Chapter 3 introduces the system structure of the adaptive schemes proposed in this thesis. In this structure, there are two control components: a non-adaptive computed torque control and an adaptive control. For the tracking error dynamics obtained by the computed torque scheme, quite large tracking errors will remain if the a priori estimates employed by this scheme do not match the true system parameters very well (which is the usual situation) and the error dynamics do not form a free system as in the ideal case. In order to reduce the tracking errors, the second control – adaptive component – will be introduced to make further compensation. This system structure sets up the basis for the adaptive controller designs developed in Chapter 4 and 5.

Based on the system structure obtained in Chapter 3, the first adaptive control algorithm is proposed in Chapter 4. To reduce the tracking error resulting from the pure computed torque method, the adaptive controller design is based on a decentralized error system configuration with uncertainties and interconnections among error subsystems. The Lyapunov direct method is used in controller design and quantitative boundedness of the position errors is obtained. The acceleration measurements are avoided by introducing filter operators.

In Chapter 5, another adaptive control algorithm, which is an enchancement of the method given in Chapter 4, is developed. A linear operator is introduced to avoid the need for acceleration measurements and to obtain a strictly positive real state space realization. Then the relationships between the magnitudes of the interconnections and the magnitudes of the overall system positions and velocities are investigated. Based on the results obtained, the second adaptive control algorithm is developed using the decentralized system adaptive controller design method. Theorem 5-1, in this chapter, shows the boundedness of overall system tracking errors and the parameter estimation errors and their convergence rates. The result on the bounded adaptive control torque is also obtained.

In order to verify the algorithms presented in Chapters 4 and 5, a case study of a SCARA robotic manipulator is presented in Chapter 6. Using this example, the controller design procedures are discussed. Moreover, utilizing a dynamic model of this robot some simulation results of proposed adaptive control algorithms are shown. The performances of these algorithms are also compared with the results of the computed torque scheme.

Finally in Chapter 7, conclusions are presented and comments on further research are given.

Chapter 2.

ROBOT DYNAMICS AND CONTROL

2-1. INTRODUCTION

In order to study robot motion control, the dynamic equations governing the motion of industrial robots will be introduced. The robots under consideration are open kinematic chain, rigid manipulators with revolute joints. For this kind of robot there are two kinds of formulations commonly used to describe the dynamics equations: Newton-Euler expressions and Lagrangian expressions. In the Lagrangian formulation, the dynamics are given in terms of work and energy using generalized coordinates. In this dissertation, the Lagrangian formulae will be employed because they are generally compact and provide a closed form expression using joint torques and joint displacements. The resulting equations of motion are a set of coupled second order nonlinear differential equations which are used in much of the literature on robot dynamics analysis.

In order to control robot systems properly, it is necessary to have a good understanding of the physical features of robot dynamics. To achieve this, some properties of the dynamic equations, which play very important roles in this study, will be stated. The properties will be illustrated by an example. Further, some assumptions are made on this system which are used throughout this thesis.

For the equations of motion obtained, the problem of motion control will be defined. As motion control normally consists of two problems: set point control (position regulation) and tracking control (trajectory following), a PD control law and its stability analysis (given in [2]) will be introduced. This analysis will explain why the joint–independent PD law works well in robot control. It is also helpful in understanding the tracking control problem. Secondly, the tracking control problem, which is the objective of the thesis, will be discussed. The computed torque scheme which is the foundation of the adaptive control methods proposed in Chapters 4 and 5 is emphasized. Furthermore, the necessity for adaptive control will be explained and an adaptive control scheme presented by Craig [12][13] will be introduced.

2-2. DYNAMIC EQUATION

2-2-1. Robot Dynamics Equation

The dynamics of rigid robot manipulator systems with n degrees of freedom can be described by the Lagrangian Equation of motion [62][52][14][3]:

$$\frac{d}{dt}(\partial L(q,\dot{q})/\partial \dot{q}) - \partial L(q,\dot{q})/\partial q = u, \tag{2-2-1}$$

where $\partial L(q,\dot{q})/\partial(\cdot)$ is the partial derivative of $\partial L(q,\dot{q})$ with respect to $(\cdot)$ and

$$L(q,\dot{q}) = \Psi(q,\dot{q}) - \Pi(q) \in R^1, \tag{2-2-2}$$

is the Lagrangian function of the whole robot system; q, $\dot{q} \in R^n$ are the generalized position and velocity vectors defined in the robot's joint coordinate space; $u \in R^n$ is the generalized input torque vector causing the motion of the robot's arms. In Eqn.(2-2-2), $\Psi(q,\dot{q}) = \frac{1}{2}\dot{q}^T D(q)\dot{q} \in R^1$ and $\Pi(q) \in R^1$ represent the total kinetic and total potential energy functions, respectively, of a robot with n linkages.

Substituting Eqn.(2-2-2) into Eqn.(2-2-1), noting that $\partial \Pi(q)/\partial \dot{q} = 0$, and denoting the gravitational torque $\partial \Pi(q)/\partial q = g(q) \in R^n$, Eqn.(2-2-1) then becomes

$$\frac{d}{dt}(D(q)\dot{q}) - \frac{\partial}{\partial q}\left(\frac{1}{2}\dot{q}^T D(q)\dot{q}\right) + g(q) = u. \tag{2-2-3}$$

Let

$$s(q,\dot{q}) = \frac{1}{2}S(q,\dot{q})\dot{q} = -\frac{\partial}{\partial q}\left(\frac{1}{2}\dot{q}^T D(q)\dot{q}\right) \in R^n \tag{2-2-4}$$

then Eqn.(2-2-3) becomes

$$\frac{d}{dt}(D(q)\dot{q})+\frac{1}{2}S(q,\dot{q})\dot{q}+g(q)=u, \tag{2-2-5a}$$

or

$$\frac{d}{dt}(D(q)\dot{q})+s(q,\dot{q})+g(q)=u. \tag{2-2-5b}$$

Since

$$\frac{d}{dt}(D(q)\dot{q}) = D(q)\ddot{q}+\dot{D}(q)\dot{q},$$

Eqn.(2-2-5a) can be rewritten as

$$D(q)\ddot{q}+\dot{D}(q)\dot{q}+\frac{1}{2}S(q,\dot{q})\dot{q}+g(q)=u.$$

Denoting

$$h(q,\dot{q})=\dot{D}(q)\dot{q}+\frac{1}{2}S(q,\dot{q})\dot{q}$$

$$=\dot{D}(q)\dot{q}-\frac{\partial}{\partial q}\left(\frac{1}{2}\dot{q}^{T}D(q)\dot{q}\right) \tag{2-2-6}$$

results in

$$D(q)\ddot{q}+h(q,\dot{q})+g(q)=u. \tag{2-2-7}$$

Moreover $h(q,\dot{q})$ can be expressed as

$$h(q,\dot{q})=H(q,\dot{q})\dot{q} \tag{2-2-8}$$

and Eqn.(2-2-7) can be rewritten as

$$D(q)\ddot{q}+H(q,\dot{q})\dot{q}+g(q)=u. \tag{2-2-9}$$

It is also possible to describe the i-th equation of Eqn.(2-2-9) by [59]

$$\sum_{j=1}^{n} d_{ij}(q)\ddot{q}_j+\sum_{j=1}^{n}\sum_{k=1}^{n} h_{ijk}(q)\dot{q}_i\dot{q}_j+g_i(q)=u_i \tag{2-2-9a}$$

2-2-2. Some Important Properties

The motion equations Eqn.(2-2-7) or Eqn.(2-2-9) have the following properties:

Property–1) $D(q)\in R^{nxn}$ is the inertia matrix of the robot system, which is positive definite i.e., $D(q)=D^T(q)>0$ for all q. This also implies that D(q) is non-singular and its inverse is positive definite as well. Its derivative with respect to time t exists.

Property–2) $h(q,\dot{q})=H(q,\dot{q})\dot{q}$ is the centrifugal and Coriolis force vector. According to Eqn.(2-2-6) it fully depends on the inertia matrix. In view of Eqn.(2-2-9a), it can be seen that every element of $h(q,\dot{q})$, denoted by $h_i(q,\dot{q})$, is a quadratic form in $\dot{q}$ so that $h(q,\dot{q})$ is a quadratic form in $\dot{q}$ as well. This results in a non-unique representation of $H(q,\dot{q})$, Among the different representations of $H(q,\dot{q})$, it is always possible to write it in such a way that $J=\dot{D}(q)-2H(q,\dot{q})$ is a skew symmetric matrix [2][56], i.e., it satisfies

$$J=\dot{D}(q)-2H(q,\dot{q})=-J^T. \qquad (2\text{-}2\text{-}10)$$

In this thesis, except where specifically noted, it is supposed that $H(q,\dot{q})$ satisfies Eqn.(2-2-10).

Property-3) Eqn.(2-2-7) or Eqn.(2-2-9) can be written in such a way that it is linear in the system parameters (see [5], [13] and [56])

$$D(q)\ddot{q}+H(q,\dot{q})\dot{q}+g(q)=\Omega(q,\dot{q},\ddot{q})\theta, \qquad (2\text{-}2\text{-}11a)$$

where $\theta\in R^m$ is the dynamic constant parameter vector related to the mass, inertial tensors, geometrical sizes, and the payloads of the robot arms; $\Omega(q,\dot{q},\ddot{q})\in R^{nxm}$ is such that its every element is a non-linear function of q, $\dot{q}$ and $\ddot{q}$. These nonlinear functions are determined by the geometric configuration of the robots and can be obtained by investigating the kinematics and dynamics. Moreover let $D_i(q)$ be the i-th row of D(q) and

$h_i(q,\dot{q})$ the i-th component of $h(q,\dot{q})=H(q,\dot{q})\dot{q}$, then the i-th component of Eqn.(2-2-11a) can be written as

$$D_i(q)\ddot{q}+h_i(q,\dot{q})+g_i(q)=\omega_i(q,\dot{q},\ddot{q})\theta_i. \tag{2-2-11b}$$

The positive definiteness of the inertia matrix D(q) (Property-2) can be easily understood by considering the fact that physically the kinetic energy of the motion system is always a positive value, i.e.,

$$\Psi(q,\dot{q})=\frac{1}{2}\dot{q}^T D(q)\dot{q}>0. \qquad \text{for any } \dot{q}\neq 0.$$

The Property-2) implies that Eqn.(2-2-7) or Eqn.(2-2-9) is a passive system. Calculating the derivative of the total kinetic energy function $\Psi(q,\dot{q})$ with respect to time gives:

$$\frac{1}{2}\frac{d}{dt}[\dot{q}^T D(q,\dot{q})\dot{q}] = \dot{q}^T D(q,\dot{q})\ddot{q}+\frac{1}{2}\ \dot{q}^T\dot{D}(q,\dot{q})\dot{q}.$$

Taking Eqn.(2-2-9) and Eqn.(2-2-10) into account,

$$\frac{1}{2}\frac{d}{dt}[\dot{q}^T D(q,\dot{q})\dot{q}] = \dot{q}^T[-H(q,\dot{q})\dot{q}-g(q)+u+\frac{1}{2}\dot{D}(q,\dot{q})\dot{q}]$$

$$= \dot{q}^T[-g(q)+u]$$

or

$$\frac{1}{2}[\dot{q}^T D(q,\dot{q})\dot{q}]=\int_0^t \dot{q}^T[-g(q)+u]dt,$$

which means that all the input energy provided by the input torque and the gravitational torque are converted into the system kinetic energy and the system itself neither absorbs nor yields any energy.

It should be noted that in the motion equation given above the viscous and Coulomb friction torque have been ignored. The equations correspond to the models of direct driven robots. In fact, no matter how insignificant it is, this sort of torque/force exists in most systems and represents the dissipativeness of the system. The damping effect of this torque will tend to make the system more stable. For instance, if the friction force is linear, it can be described by the term $F_c\dot{q}$ (F_c=diag{f_i} with f_i>0 for all i is the friction coefficient matrix), then Eqn.(2-2-9) becomes:

$$D(q)\ddot{q}+H(q,\dot{q})\dot{q}+g(q)+F_c\dot{q}=u. \qquad (2\text{-}2\text{-}12)$$

Further, the actuator dynamics of the robots have been ignored in the motion equation Eqn.(2-2-7) or Eqn.(2-2-9). This treatment will certainly cause some modelling errors. However since a large number of robot arms are driven by DC servo motors with fast, high-gain current loops, the transfer function of the actuator can be reasonably approximated by a constant torque gain.

For the robot dynamic equation Eqn.(2-2-7) or Eqn.(2-2-9), the following assumptions are made which will stand for all sections in this thesis:

Assumption 2-1) All joints of the robotic manipulators under consideration are revolute [13] [44].

This assumption results in some special properties of the equation of motion Eqn.(2-2-7). As all joints are revolute, each element in the inertial matrix D(q) and gravitational torque vector g(q) is made up of trigonometric functions of q so that D(q) and g(q) are all continuous and bounded by some constants related to the dynamic parameters of the equation. This implies there exist some constants c_q >0 and c_g >0 such that

$$\|D(q)\|\le c_D, \qquad (2\text{-}2\text{-}13)$$

$$\|g(q)\|\le c_g. \qquad (2\text{-}2\text{-}14)$$

Moreover, for the centrifugal and the Coriolis torque $h(q,\dot{q})$,

$$\|h(q,\dot{q})\|\le c_h\|\dot{q}\|. \qquad (2\text{-}2\text{-}15)$$

Eqn.(2-2-15) means that if the velocities of the joints are bounded, the centrifugal and the Coriolis torque will be bounded as well due to the fact that all functions of joint positions in this sort of torque are trigonometric functions of q and $h(q,\dot{q})$ is a quadratic form in $\dot{q}$.

Eqns.(2-2-13) – (2-2-15) also imply that for each element of $D(q)$, $h(q,\dot{q})$ and $g(q)$, there are some positive constants d^o_{ij}, g^o_i, and h^o_{ij}, such that

$$|D_{ij}(q)| \le d^o_{ij} \tag{2-2-16}$$

$$|g_i(q)| \le g^o_i \tag{2-2-17}$$

and

$$|h_i(q,\dot{q})| \le \sum_{j=1}^{n} h^o_{ij}|\dot{q}_j|. \tag{2-2-18}$$

In addition, in view of Eqn.(2-2-4),

$$|s_i(q,\dot{q})| \le \sum_{j=1}^{n} s^o_{ij}|\dot{q}_j|, \tag{2-2-19}$$

where s^o_{ij} are positive constants, as $|s_i(q,\dot{q})|$ is a quadratic form in $\dot{q}_i$ as well.

2-2-3. An Example

In order to illustrate the properties of robot dynamics mentioned previously, this section will look at an example [38].

The robotic manipulator used here is a SCARA robot with four degrees of freedom. Without losing generality, only its first two joints, which move horizontally, are considered. In this model all friction force torques and actuator dynamics are ignored. The structure of it is shown in Fig.(2-2-1).

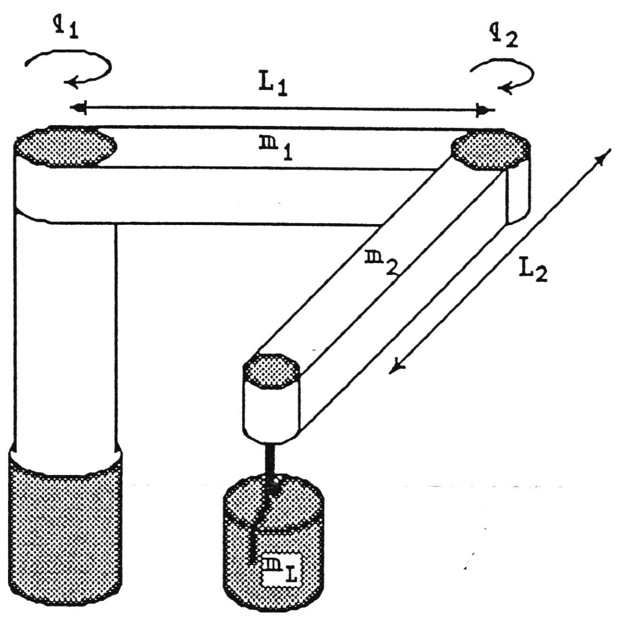

Fig.(2-2-1). The Scara robot with two horizontally moving linkages.

Assuming that the masses of the links are uniformly distributed, it can be shown that the inertial matrix is

$$D(q)=\begin{bmatrix} d_{11}(q) & d_{12}(q) \\ d_{21}(q) & d_{22}(q) \end{bmatrix}. \qquad (2\text{-}2\text{-}20)$$

In this matrix,

$$d_{11}(q) = c_1+c_3+c_4m_L+(c_2+2L_1L_2m_L)\cos q_2 \qquad (2\text{-}2\text{-}21a)$$

$$d_{12}(q) = d_{21}(q) = c_3+L_2^2m_L+(c_2/2+L_1L_2m_L)\cos q_2 \qquad (2\text{-}2\text{-}21b)$$

$$d_{22}(q) = c_3+L_2^2m_L \qquad (2\text{-}2\text{-}21c)$$

where

$$c_1 = L_1^2(m_1/3+m_2) \tag{2-2-22a}$$

$$c_2 = L_1L_2m_2 \tag{2-2-22b}$$

$$c_3 = L_2^2m_2/3 \tag{2-2-22c}$$

$$c_4 = L_1^2+L_2^2. \tag{2-2-22d}$$

In the equalities above:

L_1 (m):	length of the first link;
L_2 (m):	length of the second link;
m_L(kg):	mass of payload fixed at the end of link 2;
m_1 , m_2(kg):	masses of the first and second link respectively.

It is now shown that this robot system satisfies the Property 1 – 3 given by the previous section:

The satisfaction of Property-1, i.e., the positive definiteness of D(q), for arbitrary L_1, L_2, m_1, m_2, $m_L>0$, and q_2, can be proved by using the features of a positive definite matrix. In this case it is required that $d_{11}(q)>0$ and $|D(q)|>0$. Consider Eqn.(2-2-21a):

$$d_{11}(q) = c_1+c_3+c_4m_L+(c_2+2L_1L_2m_L)\cos q_2$$

first. As $|\cos q_2|\leq 1$, if condition

$$c_1+c_3+c_4m_L-(c_2+2L_1L_2m_L)>0, \tag{2-2-23}$$

is satisfied, then $d_{11}(q)$ must be greater than zero. Substituting dynamic parameters L_i and m_i from Eqn.(2-2-22) into the left hand side of Eqn.(2-2-23) gives

$$c_1+c_3+c_4m_L-(c_2+2L_1L_2m_L)$$

$$= L_1^2(m_1/3+m_2)+L_2^2m_2/3+(L_1^2+L_2^2)m_L-(L_1L_2m_2+2L_1L_2m_L)$$

$$= L_1^2(m_1/3+m_2)+L_2^2m_2/3+(L_1-L_2)^2m_L-L_1L_2m_2.$$

Completing the square of the first two terms yields

$$c_1+c_3+c_4m_L-(c_2+2L_1L_2m_L)$$

$$=(L_1\sqrt{m_1/3+m_2}-L_2\sqrt{m_2/3})^2+(L_1-L_2)^2m_L+2L_1L_2[(m_1/3+m_2)(m_2/3)]^{1/2}-L_1L_2m_2.$$

Obviously the first two terms are both greater than zero. For the third term, as both m_1 and $m_2>0$,

$$2L_1L_2\sqrt{(m_1/3+m_2)(m_2/3)}$$

$$=2L_1L_2\sqrt{m_1m_2/9+m_2^2/3}$$

$$> 2L_1L_2\sqrt{(m_2^2)/3}$$

$$=(2/\sqrt{3})L_1L_2m_2$$

$$>L_1L_2m_2,$$

that is, the third term is greater than the fourth term. This means the inequality given by Eqn.(2-2-23) holds and therefore $d_{11}(q)>0$.

Moreover, the determinant of the inertial matrix D(q) is

$$|D(q)|=d_{11}(q)d_{22}(q)-d_{12}^2(q).$$

In view of Eqn.(2-2-21) and Eqn.(2-2-22), this becomes

$$|D(q)|=L_1L_2(m_1m_2/9+m_1m_L/3+m_2m_L/3+m_2^2/12)>0.$$

Then, since $d_{11}(q)$ and $|D(q)|$ are both greater than zero for arbitrary L_1, L_2, m_1, m_2, $m_L>0$, and q_2, it can be seen that $D(q)$ is a positive definite matrix which satisfies Property-1.

To examine Property-2, the centrifugal and Coriolis torque vector of this robot system can be written as

$$h(q,\dot{q})=\dot{D}(q)\dot{q}-\frac{\partial}{\partial q}\left(\frac{1}{2}\dot{q}^T D(q)\dot{q}\right)=\begin{bmatrix} h_1(q,\dot{q}) \\ h_2(q,\dot{q}) \end{bmatrix}$$

according to Eqn.(2-2-6), where

$$h_1(q,\dot{q})=-2c_o\sin q_2\dot{q}_1\dot{q}_2-c_o\sin q_2\dot{q}_2\dot{q}_2, \tag{2-2-24a}$$

$$h_2(q,\dot{q})=c_o\sin q_2\dot{q}_1\dot{q}_1, \tag{2-2-24b}$$

with $c_o=c_2/2+L_1L_2m_L$. They are both quadratic forms of $\dot{q}$ as shown in Eqn.(2-2-9a). It is obvious that $H(q,\dot{q})$ in Eqn.(2-2-8) is not unique as it can be described as either

$$h(q,\dot{q})=H(q,\dot{q})\dot{q}=\begin{bmatrix} -2c_o\sin q_2\dot{q}_2 & -c_o\sin q_2\dot{q}_2 \\ c_o\sin q_2\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

or

$$h(q,\dot{q})=H(q,\dot{q})\dot{q}=\begin{bmatrix} -c_o\sin q_2\dot{q}_2 & -c_o\sin q_2(\dot{q}_1+\dot{q}_2) \\ c_o\sin q_2\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

Moreover, it is easy to prove that only the latter one satisfies property $\dot{D}(q)-2H(q,\dot{q}) = J = -J^T$ given by Eqn.(2-2-10), i.e.,

$$\dot{D}(q)-2H(q,\dot{q})=\begin{bmatrix} 0 & c_o \sin q_2(2\dot{q}_1+\dot{q}_2) \\ -c_o \sin q_2(2\dot{q}_1+\dot{q}_2) & 0 \end{bmatrix}$$

which is a skew matrix.

Before justifying Property-3, it should be noted that, in this example, due to the fact that both linkages are constrained to move in the horizontal plane, the gravitational torque g(q) is zero. For Property-3, the following expression corresponds to the linear description of parameters (see Eqn.(2-2-11a)):

$$D(q)\ddot{q}+h(q,\dot{q})=\begin{bmatrix} \ddot{q}_1 & \cos q_2\ddot{q}_1 & \ddot{q}_2 & \cos q_2\ddot{q}_2-\sin q_2\dot{q}_2(2\dot{q}_1+\dot{q}_2) \\ 0 & 0 & 2\ddot{q}_2 & \sin q_2\dot{q}_1\dot{q}_1+\cos q_2\ddot{q}_2 \end{bmatrix}\begin{bmatrix} c_1+c_3+c_4m_L \\ c_2+2L_1L_2m_L \\ c_3+L_2L_2m_L \\ c_2/2+L_1L_2m_L \end{bmatrix}$$

It is also possible to show that each subsystem can be expressed as being linear in its parameters (see Eqn.(2-2-11b)). For instance, in this case, subsystem 2 is given by

$$D_2(q)\ddot{q}+h_2(q,\dot{q})=\omega_2(q,\dot{q},\ddot{q})\theta_2,$$

where

$$\omega_2(q,\dot{q},\ddot{q})=[\ 2\ddot{q}_2 \quad \sin q_2\dot{q}_1\dot{q}_1+\cos q_2\ddot{q}_2\ ]$$

$$\theta_2^T=[\ c_3+L_2L_2m_L \quad c_2/2+L_1L_2m_L\ ].$$

Moreover, as both joints of the arm are revolute, this example also satisfies Assumption 2-1 given by the previous section. It can be seen, by looking at Eqn.(2-2-21), that the elements of D(q) are all made up of trigonometric functions of the position q_2, and therefore $d_{ij}(q)$ are all bounded functions for any q. The quadratic forms in $\dot{q}$ of $h_i(q,\dot{q})$ in Eqns.(2-2-24a,b) verify that $h_i(q,\dot{q})$ will be bounded if $\dot{q}$ is bounded. This means that there are constants c_D and $c_h > 0$ such that Eqns.(2-2-13) and (2-2-15) are both satisfied.

2-3. MOTION CONTROL

Among the various motion control problems of robotic manipulators, position control and trajectory tracking are two of the fundamentally important problems. In the application of position control, such as spot welding and material handing, the robot grippers are required to move from one position to another regardless of the trajectories they follow. In these cases what is of concern is the position accuracy rather than the transient response. However, for tasks such as spray painting, laser cutting, or where there are some obstacles in the robot's working envelope, the robots have to follow certain trajectories to give a uniform spray coat, precise cutting edges, or to avoid collisions with obstacles. In these sorts of applications the whole trajectory can be divided into many short intervals and in each of them the robot arms can be moved by a position control scheme. However, this will not control the transient responses and the velocities will be slowed. In these cases trajectory tracking motion control is needed to give good performance in following the given trajectories.

2-3-1. The PD Position Control Law

It is a common observation that most contemporary robots present quite good performance in position control using joint independent non-model based control strategies such as PD or PID control laws. It is natural to ask why this simple non-model based control law can handle complicated non-linear dynamics of controlled robots. Using the passivity property of robot mechanical systems, Aromoto and Miyazaki, in [2], discovered that robots are one class of passive mechanical systems if friction can be ignored. It is this property that makes them work well with set point control schemes.

In set point control, the PD control law is given by [2]:

$$u=u_d+u_g$$

$$= -K_v\dot{q} +K_p(q_{do}-q)+g(q), \qquad (2\text{-}3\text{-}1)$$

where q_{do} is a constant, defining a set point in the joint space for the robot to achieve, $K_v=\mathrm{diag}\{k_{vi}\}\in R^{nxn}$, $K_p=\mathrm{diag}\{k_{pi}\}\in R^{nxn}$ with k_{vi}, $k_{pi}>0$ for i=1,2,...n,

$$u_d= -K_v\dot{q} +K_p(q_{do}-q)$$

is the negative velocity feedback and position error feedback law, and

$$u_g= g(q),$$

is the gravitational torque compensation term. Applying Eqn.(2-3-1) to Eqn.(2-2-9), the closed-loop system becomes

$$D(q)\ddot{q}+H(q,\dot{q})\dot{q}+K_v\dot{q}+K_p(q-q_{do})=0. \qquad (2\text{-}3\text{-}2)$$

It should be noted that as q_{do} is a constant, Eqn.(2-3-2) represents an autonomous equation. As shown in [2], for this autonomous closed loop system, a candidate for the Lyapunov function

$$v(q,\dot{q})=\frac{1}{2}\dot{q}^T D(q)\,\dot{q}+\frac{1}{2}(q-q_{do})^T K_p(q-q_{do})$$

can be introduced, which is positive. Its total derivative is

$$\dot{v}(q,\dot{q})=\dot{q}^T D(q)\,\ddot{q}+\frac{1}{2}\dot{q}^T\dot{D}(q)\,\dot{q}+\dot{q}^T K_p(q-q_{do})$$

$$=\dot{q}^T[-H(q,\dot{q})\dot{q}-g(q)+u+\frac{1}{2}\dot{D}(q)\,\dot{q}+K_p(q-q_{do})].$$

If the PD control law given by Eqn.(2-3-1) is substituted into the equation above, and since that $\frac{1}{2}\dot{q}^T\dot{D}(q)\dot{q}-\dot{q}^T H(q,\dot{q})\dot{q}=0$ according to Property-2 in Section 2-2-2 (see Eqn.(2-2-10)), then $\dot{v}(q,\dot{q})$ becomes

$$\dot{v}(q,\dot{q})=-\dot{q}^T K_v \dot{q}.$$

Since K_v is positive definite, $\dot{v}(q,\dot{q})<0$. This means that $\dot{q}$ is globally asymptotically convergent to zero as $t\rightarrow\infty$ and q will be bounded if q(0) and $\dot{q}(0)$ are both bounded. Moreover $\dot{q}\rightarrow 0$ implies q tends to a certain point q_c=constant inside the joint space. According to the invariant principle of LaSalle [31], the solution trajectory $(\dot{q}(t),q(t))$ of equation Eqn.(2-2-7) will approach and stay within the set

$$E=\{x=(\dot{q},q)\in X:\ \dot{v}=0\}$$

as the system is autonomous, where X is the joint space of the robot. This means in set E, $\dot{q}\equiv 0$ and therefore $\ddot{q}\equiv 0$. Then Eqn.(2-3-2) becomes

$$K_p(q-q_{do})=0.$$

As K_p is positive definite, so it must have $q_c=q_{do}$ which means $q\rightarrow q_{do}$ as $t\rightarrow\infty$, that is, the PD control law Eqn.(2-3-1) gives zero position error.

If a given robot's friction term is big enough and cannot be ignored (as is the case for most non-direct drive robots), its equation can be given by rewriting Eqn.(2-2-12) as:

$$D(q)\ddot{q}+H(q,\dot{q})\dot{q}+g(q)=u_o-F_c\dot{q}, \qquad (2\text{-}3\text{-}3)$$

if the friction torque is linear. From the stability analysis given above and by comparing the system equation including the friction torque Eqn.(2-2-12) with Eqn.(2-3-2), it can be seen that the effect of introducing velocity feedback is the same as increasing the friction term in Eqn.(2-2-9), which actually makes the system dissipative. This also implies, according to the analysis of stability of system Eqn.(2-3-2), that even by means of pure position feedback control law and gravitational force compensation

$$u_o=K_p(q_d-q)+g(q),$$

the system will be stable and will give zero position error because the friction coefficient matrix F_c is a positive definite matrix.

Physically, stability of the PD controller relies on two facts: i) the inherent passivity of robot systems themselves and ii) negative velocity feedback and gravitational compensation of the control law which increase the damping effect in closed loop systems so that the system states are stable at their equilibriums.

2-3-2. Tracking Control

In the problem of tracking control, the robots are required to follow a moving trajectory rather than achieve a set point in the joint space. In this case, the PD control will not give good tracking performance and the model based control method is needed.

In tracking control, it is assumed that the trajectories for the robot end-effectors to follow, described in the Cartesian space, have been mapped into the joint space of the robot arms. For these trajectories the following assumption is made:

Assumption 2-2) The reference trajectory for the robot to follow is given by $q_d=q_d(t)$ which is a uniform continuous function of time t. It is also assumed that $\dot{q}_d$ and $\ddot{q}_d$ exist .

The tracking control problem can be defined as follows: for a given robot which is characterized by its dynamic equation, and a given trajectory specified by q_d, $\dot{q}_d$ and $\ddot{q}_d$ satisfying the assumption A-2-2), it is necessary to synthesize a control torque u so that the robot state can follow the reference trajectory as closely as possible. If $q_d \equiv q_{do}$ is a set point in the robot joint space, then a point to point control problem is defined.

According to how much knowledge of the robot dynamics are used, the control schemes can be classified as being either non-model based or model based. For instance, the joint independent PD control represents a non-model based method and the Computed Torque scheme is a model based one.

In this thesis the well known computed torque scheme is concentrated on. As mentioned in Section 1-2, this control law is based on the inverse dynamics of the controlled robot motion equation:

$$u=D(q)(\ddot{q}_d+K_v(\dot{q}_d-\dot{q})+K_p(q_d-q))+h(q,\dot{q})+g(q). \tag{2-3-4}$$

where $K_v=\mathrm{diag}\{k_{vi}\}\in R^{nxn}$, $K_p=\mathrm{diag}\{k_{pi}\}\in R^{nxn}$ with k_{vi}, $k_{pi}>0$ for $i=1,2,...n$.

Applying this control law to the equation of motion Eqn.(2-2-7), tracking error dynamics will be obtained, given by

$$\ddot{e}+K_v\dot{e}+K_pe=0 \tag{2-3-5}$$

where $e=q_d-q$ is the tracking error vector. As this is a free system, by proper choice of K_p and K_v in Eqn.(2-3-4) the tracking error e will converge to zero for any initial conditions if $D(q)$, $h(q,\dot{q})$ and $g(q)$ in Eqn.(2-3-4) match those in the system dynamics Eqn.(2-2-7) exactly. In [7] and [65], Wen and Bayard proved that the control law Eqn.(2-3-4) guarantees exponential convergence for the tracking errors.

2-4. ADAPTIVE CONTROL

As shown previously, in order to ensure stability and good performance in tracking control, model based control schemes are needed. However, the implementation of these methods is limited by the fact that in some circumstances it is almost impossible to obtain a precise model for given robots because of their complicated system dynamics. To deal with this problem, adaptive control methods provide potential solutions because of their ability to improve their knowledge of unknown or partly unknown dynamics of the controlled system (by on-line estimation) and adjust their control functions (by selfadjustment mechanisms) to ensure the stability of the overall system and to achieve certain indexes of performance.

Among the various adaptive control schemes available, the method proposed by Craig [12] and [13] will be introduced here as the schemes proposed in this thesis use these as a starting point.

Consider the equation of motion Eqn.(2-2-7)

$$D(q)\ddot{q}+h(q,\dot{q})+g(q)=u, \tag{2-2-7}$$

and suppose that all nonlinear functions of q and $\dot{q}$ in each element of D(q), $h(q,\dot{q})$ and g(q) are known and the dynamic parameters of Eqn.(2-2-7) are unknown or partly known but there are some estimates on these parameters available. In this case the adaptive control law is given by

$$u=\hat{D}(q)[\ddot{q}_d+K_v(\dot{q}_d-\dot{q})+K_p(q_d-q)]+\hat{h}(q,\dot{q})+\hat{g}(q). \tag{2-4-1}$$

In (2-4-1), $(\hat{\cdot})$ expresses an estimate of $(\cdot)$ which is obtained by replacing the unknown dynamic parameters by their estimates. This results in error dynamics if $\hat{D}^{-1}(q)$ exists:

$$\ddot{e}+K_v\dot{e}+K_pe=\hat{D}^{-1}(q)[\tilde{D}(q)\ddot{q}+\tilde{h}(q,\dot{q})+\tilde{g}(q)] \tag{2-4-2}$$

where $\tilde{D}(q)=D(q)-\hat{D}(q)$, $\tilde{h}(q,\dot{q})=h(q,\dot{q})-\hat{h}(q,\dot{q})$ and $\tilde{g}(q)=g(q)-\hat{g}(q)$ are the estimation errors of D(q), $h(q,\dot{q})$ and g(q) respectively. Comparing this with Eqn.(2-3-5), it can be seen that due to the existence of dynamic parameter errors in control law Eqn.(2-4-1), the tracking error dynamics Eqn.(2-4-2) is no longer a free system and the characteristics of e will depend upon the right hand side of it.

In accordance with the fact that all of the nonlinear functions in q and $\dot{q}$ in D(q), $h(q,\dot{q})$ and g(q) are known and Eqn.(2-2-11a) (Property-3), Eqn.(2-4-2) can be rewritten as

$$\ddot{e}+K_v\dot{e}+K_pe=\hat{D}^{-1}(q)\Omega(q,\dot{q},\ddot{q})\tilde{\theta}. \tag{2-4-3}$$

Let $x=[e \;\; \dot{e}]^T$, then Eqn.(2-4-3) above can be expressed by the state space equation

$$\dot{x}=Ax+B\hat{D}^{-1}(q)\Omega(q,\dot{q},\ddot{q})\bar{\theta}, \tag{2-4-4}$$

where

$$A=\begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix} \quad \text{and} \quad B=\begin{bmatrix} 0 \\ I \end{bmatrix}.$$

As system matrix A is stable, according to Lyapunov stability theory, for a positive definite matrix Q, there exists a positive matrix P satisfying the following Lyapunov equation

$$A^TP+PA=-Q. \tag{2-4-5}$$

For this error state space equation, the adaptive control law can be derived by the Lyapunov direct method. A candidate for the Lyapunov function is given by

$$v(x,\bar{\theta}) =\frac{1}{2}x^TPx+\frac{1}{2}\bar{\theta}^T\Gamma^{-1}\bar{\theta}, \tag{2-4-6}$$

where $P=P^T>0$ and $\Gamma=\text{diag}\{\gamma_1 \; \gamma_2 \; ... \; \gamma_m\}$ with $\gamma_i>0$ for all $i=1,2,...,m$. The scalar function $v(x,\bar{\theta})$ is positive definite for any x, $\bar{\theta}\neq 0$. Its total derivative along the solution trajectory of Eqn.(2-4-4) is

$$\dot{v}(x,\bar{\theta})= -\frac{1}{2}x^T(A^TP+PA)x+ x^T PB\hat{D}^{-1}(q)\Omega(q,\dot{q},\ddot{q})\bar{\theta}+ \dot{\bar{\theta}}^T\Gamma^{-1}\bar{\theta}.$$

Let the adaptation law be

$$\dot{\bar{\theta}} = -\Gamma\, \Omega^T(q,\dot{q},\ddot{q})\hat{D}^{-1}(q)B^TPx, \tag{2-4-7}$$

then it follows that

$$\dot{v}(x,\bar{\theta})= -\frac{1}{2}x^TQx. \tag{2-4-8}$$

Eqns.(2-4-6) and (2-4-8) show that for the bounded $x(0)$ and $\bar{\theta}(0)$, the $x(t)$ and, $\bar{\theta}(t)$ will be bounded. Moreover, according to [1] and [47], if the persistent excitation condition on $\Omega^T(q,\dot{q},\ddot{q})\hat{D}^{-1}(q)$

$$\alpha I \le \int_{t_o}^{t_o+\rho} (\Omega^T\hat{D}^{-1})(\Omega^T\hat{D}^{-1})^T\, dt \le \beta I$$

is satisfied for all t_o, where α, β, and $\rho>0$ are some constants, then the parameter estimate error will tend to zero, i.e., $\bar{\theta}(t)\to 0$ as $t\to\infty$. Hence, in view of Eqns.(2-4-6) and (2-4-8), it follows that $x(t)\to 0$ as $t\to\infty$, that is, both position and velocity tracking errors e and $\dot{e}$ will converge to zero.

As $\bar{\theta}= \theta-\hat{\theta}$, where θ is a constant vector, the adaptive control law Eqn.(2-4-7) can also be written as

$$\dot{\hat{\theta}} = \Gamma\, \Omega^T(q,\dot{q},\ddot{q})\hat{D}^{-1}(q)B^T Px. \qquad (2\text{-}4\text{-}9)$$

This parameter adaptive law is used to update the computed torque controller Eqn.(2-4-1), and if the parameter estimates converge to their true values, the control law will equal Eqn.(2-3-4) which is an ideal computed torque control. The adaptive control system configuration is shown in Fig. 2-1.

The key point of this adaptive controller is the utilization of Property-3 of the robot dynamics, i.e., the equations of motion are written in such a way that they are linear in the dynamic parameters. By this formulation the design method of linear adaptive reference model following systems can be applied directly and the controller has a very simple and clear structure.

However, there are three points to emphasize for control law Eqn.(2-4-7) (or Eqn.(2-4-9)):

i). Since $\hat{D}^{-1}(q)$ appears in the control law Eqn.(2-4-7), the inverse of the estimated inertia matrix which depends on $\hat{\theta}$ is required. This will make the computation complicated when the number of robot joints increase;

ii). In addition to i), the parameter adaptation law must ensure that the estimate of the unknown dynamic parameters are close enough to their true values so that $\hat{D}(q)$ is always an non-singular matrix. For this, Craig suggested a "cut off" function to the adaptation law. That is, it is supposed that as the bounds of the unknown parameters are known, then the estimator should stop updating as soon as the estimates achieve these bounds and let the estimates remain at their old values to make sure that $\hat{D}$ is always non-singular. This requires a good a priori knowledge of the unknown dynamic parameters which in some circumstances is difficult to achieve.

iii). In the implementation of $\Omega^T(q,\dot{q},\ddot{q})$, in addition to q, $\dot{q}$, the accelerations of the robot arm joints are also required to be measured. This may cause some technical problems as the direct measurement of the accelerations is likely to introduce a large amount of noise into the systems.

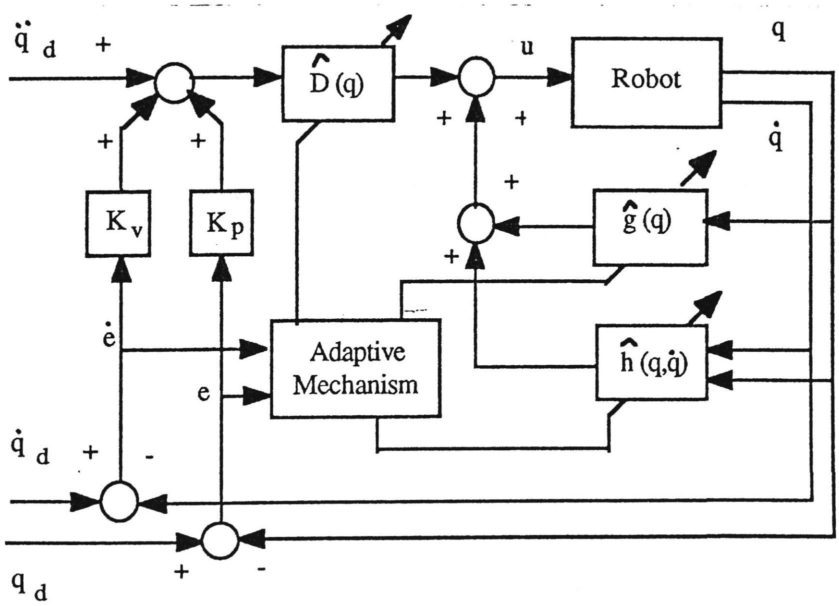

Fig. 2-1. Robot motion adaptive control system.

2-5. SUMMARY

This chapter gives a general introduction to the equations of motion for the robot dynamic systems and two basic motion control problems: position control and trajectory following control.

For the equations of motion, in Section 2-1, the Lagrangian description of the equation of motion was introduced. The system equations are a set of second order nonlinear differential equations with coupling between the joints. In spite of the complexity of the equations, some important properties provide an insight into their physical behaviour. These properties were discussed in this section. The positive definiteness of the inertial matrix and the linear-in-parameters formulation of the equation of motion are two important properties used in the following chapters. Furthermore, in order to give an interpretation of the properties an example was illustrated.

In Section 2-3, two types of basic motion control problems – position control and trajectory following control – are mentioned. In position control, the robust property of the joint independent PD control was reviewed. For trajectory following control, the computed torque scheme was introduced which also serves as a foundation in the adaptive control scheme proposed in Chapter 3. As this method is a model based scheme, its practical applications are limited by the difficulties of obtaining precise dynamic models for the given robot systems due to the their complexities. In order to overcome this problem adaptive control can be used. In Section 2-4, Craig's method was introduced.

Chapter 3.

DECENTRALIZED ERROR SYSTEM STRUCTURE

3-1. INTRODUCTION

As mentioned before, due to the complexity of robot dynamics, it is difficult to have precise prior knowledge of some dynamic parameters such as the masses and the inertia tensors for a given robot arm. This limits the applications of model based control schemes. Adaptive control schemes provide a solution to this problem because of their abilities in real time estimation of the unknown dynamic parameters and in making self-adjustments. This chapter and the following chapters will be devoted to the investigation of novel adaptive control methods.

This chapter is about the system structure for the adaptive controller design. At first, the system parameterization of the equations of motion will be discussed. The parameterization is based on the property that the equation of motion can be formulated by linear expressions in the dynamic parameters. It will be shown that by means of this formulation, the linear adaptive method can be used directly in robot motion control.

Secondly, based on the reasonable assumption that all nonlinear functions of the system state are known, the two component control law structure is proposed. The first one, as a non-adaptive control component, is the computed torque law based on an a priori estimate of the system's unknown parameters and the second one is an adaptive control component which will be used to further compensate the tracking errors.

For the error dynamics obtained by the application of the non-adaptive control component, the overall system is treated as n separate subsystems with interconnections between each other. The advantages of this treatment are that each subsystem appears as a scalar equation which only requires the diagonal elements of the inverse of the inertia matrix and gives more freedom to the design of the adaptive control law.

The determination of the adaptive control laws will be discussed in Chapters 4 and 5.

3-2. SYSTEM EQUATION AND ASSUMPTIONS

Consider the equation of motion Eqn.(2-2-7) again,

$$D(q)\ddot{q}+h(q,\dot{q})+g(q)=u, \qquad (3\text{-}2\text{-}1)$$

where, according to Eqn.(2-2-6),

$$h(q,\dot{q})=\dot{D}(q)\dot{q}+s(q,\dot{q}), \qquad (3\text{-}2\text{-}2a)$$

$$s(q,\dot{q})=-\frac{\partial}{\partial q}\left(\frac{1}{2}\dot{q}^T D(q)\dot{q}\right). \qquad (3\text{-}2\text{-}2b)$$

For this motion equation an uncertainty term $d_o(t)=[d_{o1}, d_{o2},\ldots,d_{on}]\in R^n$ is introduced, which may include frictional torques and coupling torques ignored in the modelling, disturbance torques from the environment, measurement noise, payload variations, and ignored actuator dynamics etc.. Thus, Eqn.(3-2-1) becomes

$$D(q)\ddot{q}+h(q,\dot{q})+g(q)+d_o=u, \qquad (3\text{-}2\text{-}3)$$

which is a standard Lagrangian formulation.

3-2-1. System Parameterization

According to Property-3) given in Section 2-2, nonlinear equation Eqn.(3-2-3) can be expressed as an equation which is linear in the system parameters. Thus, D(q) can be written as

$$D(q)=\{D_{ij}(q)\}=\{\sum_{k=1}^{m_{ij}} d_{ijk}f_{dijk}(q)\}. \qquad (3\text{-}2\text{-}4a)$$

Similarly, $h(q,\dot{q})$, $s(q,\dot{q})$ and $g(q)$ can be represented by

$$h(q,\dot{q}) = \{h_i(q,\dot{q})\}=\{\sum_{k=1}^{m_{ih}} h_{ik}f_{hik}(q,\dot{q})\}, \tag{3-2-4b}$$

$$s(q,\dot{q}) = \{s_i(q,\dot{q})\}=\{\sum_{k=1}^{m_{ik}} s_{ik}f_{sik}(q,\dot{q})\}, \tag{3-2-4c}$$

and

$$g(q)=\{g_i(q)\} =\{\sum_{k=1}^{m_{ig}} g_{ik}f_{gik}(q)\} \tag{3-2-4d}$$

where m_{ij}, m_{ih}, m_{ik} and $m_{ig}\geq 1$ are integers; d_{ijk}, h_{ik}, s_{ik} and g_{ik} are constant parameters related to the masses, inertia of the linkages and payload of the robot arm; $f_{dijk}(q)$, $f_{gik}(q)$, $f_{hik}(q,\dot{q})$ and $f_{sik}(q,\dot{q})$ are nonlinear functions in q and $\dot{q}$. It is worth noting that these functions are only determined by the geometrical configurations of the robots and therefore can be worked out by kinematics investigations. From Eqns.(3-2-2a,b), it can be shown that d_{ijk}, h_{ik}, and k_{ik} are all dependent on D(q). As shown in Section 2-2, the i-th equation of Eqn.(3-2-3) can be written as

$$D_i(q)\ddot{q}+h_i(q,\dot{q})+g_i(q)+d_{oi}(t)=\theta_i^T\omega_i(q,\dot{q},\ddot{q}). \tag{3-2-5}$$

where

$$\theta_i^T=[d_{i11},..d_{i1m_i},d_{i21},..d_{i2m_i},..d_{in1},...d_{inm_i},..,h_{i1},..h_{im_{ih}},g_{i1},..g_{im_{ig}}]$$

$$\omega_i^T(q,\dot{q},\ddot{q})=[f_{i11}(q)\ddot{q}_1,...,f_{i1m_i}(q)\ddot{q}_1,...,f_{in1}(q)\ddot{q}_n,...,f_{inm_i}(q)\ddot{q}_n,$$

$$f_{hi1}(q,\dot{q}),..., f_{him_{ih}}(q,\dot{q}), f_{gi1}(q), ..., f_{gim_{ig}}(q)].$$

It is also known that some elements of θ_i and those of θ_j may be correlated as stated in Section 2-2 and shown by the example in Section 2-2-3.

3-2-2. Assumptions

Assumption 2-1) made in Section 2-2 claimed that all joints of the robots under control are revolute so that all nonlinear functions $f_{dijk}(q)$ and $f_{gik}(q)$ in Eqn.(3-4) are bounded and continuous in q. Assumption 2-2) made in Section 2-3 then defined a class of reference trajectories used in motion control which should be sufficiently smooth. In addition to these, the following assumptions are also made:

Assumption 3-1) The nonlinear function vector $\omega_i(q,\dot{q},\ddot{q})$ in Eqn.(2-2-11b) is known;

Assumption 3-2) For the constant coefficients d_{ijk}, h_{ik} and g_{ik} in Eqns.(3-2-4) there exist a priori estimates, denoted by $\hat{d}_{ijk}$, $\hat{h}_{ik}$ and $\hat{g}_{ik}$, such that

$$\hat{D}(q)=\{\hat{D}_{ij}(q)\}=\{\sum_{k=1}^{m_{ij}} \hat{d}_{ijk}f_{dijk}(q)\} \tag{3-2-6a}$$

$$\hat{h}(q,\dot{q}) = \{\hat{h}_i(q,\dot{q})\}=\{\sum_{k=1}^{m_{ih}} \hat{h}_{ik}f_{hik}(q,\dot{q})\} \tag{3-2-6b}$$

and

$$\hat{g}(q)=\{\hat{g}_i(q)\} =\{\sum_{k=1}^{m_{ig}} \hat{g}_{ik}f_{gik}(q)\}. \tag{3-2-6c}$$

Assumption 3-3) The estimate Eqn.(3-2-6a) results in a positive definite $\hat{D}(q)$.

Assumption 3-4) The Eqn.(3-2-2a) still holds for the estimate Eqn.(3-2-6a, b), i.e., it has

$$\hat{h}(q,\dot{q})=\dot{\hat{D}}(q)\dot{q}+\hat{s}(q,\dot{q})=\dot{\hat{D}}(q)\dot{q}-\frac{\partial}{\partial q}\left(\frac{1}{2}\dot{q}^T\hat{D}(q)\dot{q}\right). \tag{3-2-7}$$

Assumption 3-5) The uncertainty term d_o is bounded by a known constant $c>0$, i.e.,

$$\|d_o(t)\|\leq c, \qquad \text{for } i=1, 2,\ldots, n. \tag{3-2-8}$$

It is worth noting that Assumptions 3-2) and 3-3) imply that the estimates given above also satisfy Eqns.(2-2-16) – (2-2-19), that is, there exist some constants d^*_{ij}, h^*_{ik}, and g^*_i such that

$$\hat{D}_{ij}(q) \le d^*_{ij} \tag{3-2-9}$$

$$\hat{h}_i(q,\dot{q}) \le \sum_{k=1}^{n} h^*_{ik} |\dot{q}_k| \tag{3-2-10}$$

and

$$\hat{g}_i(q) \le g^*_i. \tag{3-2-11}$$

3-3. NON-ADAPTIVE CONTROL AND SYSTEM STRUCTURE

In this section, the controller and overall system structures will be presented. The control law used here is a standard computed torque law plus an adaptive control component used to eliminate the tracking errors caused by the pure computed torque scheme. Unlike one-component adaptive control laws in which a single control component is updated by the on-line parameter estimate directly, this two-component structure works in such a way that a priori estimates of the robot dynamic parameters used by the non-adaptive control law (i.e., the computed torque scheme) are fixed and the estimator only updates the parameters of the adaptive controller component alone. The particular advantage of this structure is that it avoids the restriction required by one-component structures so that during real time control, the estimators must ensure that the estimated $\hat{D}(q)$ is non-singular. In the other words, by this two-component structure if the initial estimates $\hat{d}_{ijk}$ are set up properly such that $\hat{D}(q)$ is positive definite then $\hat{D}(q)$ will remain positive definite thereafter. Practically, it is always possible to find a set of $\hat{d}_{ijk}$ satisfying this condition by the study of the mechanical structures of the particular robot manipulators under control.

3-3-1. Non-Adaptive Control Law

Utilizing a priori estimates of the system parameters given by Eqn.(3-2-6a, b and c), the control law u is given by:

$$u=u_1+u_2, \tag{3-3-1a}$$

where

$$u_1=\hat{D}(q)[\ddot{q}_d+K_v(\dot{q}_d-\dot{q})+K_p(q_d-q)]+\hat{h}(q,\dot{q})+\hat{g}(q) \tag{3-3-1b}$$

is the computed torque control law, which is a non-adaptive component, and

$$u_2=\hat{D}(q)u_a \tag{3-3-1c}$$

is an adaptive control component. In the equalities above, $K_v=\mathrm{diag}\{k_{vi}\}\in R^{nxn}$, $K_p=\mathrm{diag}\{k_{pi}\}\in R^{nxn}$ with k_{vi}, $k_{pi}>0$, for i=1,2,...,n, and u_a, being a nonlinear function of q, $\dot{q}$, q_d, and $\dot{q}_d$, will be determined in Chapter 4 and 5. Obviously, Eqn.(3-3-1a) can be written as:

$$u=\hat{D}(q)[\ddot{q}_d+K_v(\dot{q}_d-\dot{q})+K_p(q_d-q)+u_a]+\hat{h}(q,\dot{q})+\hat{g}(q). \tag{3-3-1d}$$

Since the non-adaptive control law Eqn.(3-3-1b) is implemented by a priori estimates $\hat{\theta}_i$=constant, for i=1,2,...,n, the parameter estimation errors $\bar{\theta}_i=\theta_i-\hat{\theta}_i$ caused by the control Eqn.(3-3-1b) are unknown constants.

Substitution of Eqn.(3-3-1d) into Eqn.(3-2-3), gives:

$$\hat{D}(q)[\ddot{q}_d+K_v(\dot{q}_d-\dot{q})+K_p(q_d-q)+u_a]=D(q)\ddot{q}+h(q,\dot{q})-\hat{h}(q,\dot{q})+g(q)-\hat{g}(q)+d_o.$$

Substracting term $\hat{D}(q)\ddot{q}$ from both sides of the equality leads to

$$\hat{D}(q)[\ddot{q}_d-\ddot{q}+K_v(\dot{q}_d-\dot{q})+K_p(q_d-q)+u_a]$$

$$=D(q)\ddot{q}-\hat{D}(q)\ddot{q}+h(q,\dot{q})-\hat{h}(q,\dot{q})+g(q)-\hat{g}(q)+d_o.$$

Premultiplying both sides of the previous equation by $\hat{D}^{-1}(q)$, which is the inverse of $\hat{D}(q)$, and then moving u_a to the right hand side, results in:

$$\ddot{e}+K_v\dot{e}+K_pe=\hat{D}^{-1}(q)[\bar{D}(q)\ddot{q}+\bar{h}(q,\dot{q})+\bar{g}(q)+d_o]-u_a, \qquad (3\text{-}3\text{-}2)$$

where $e = q_d - q$ is the position error vector; $\bar{D}(q) = D(q)-\hat{D}(q)$, $\bar{h}(q,\dot{q}) = h(q,\dot{q})-\hat{h}(q,\dot{q})$ and $\bar{g}(q)=g(q)-\hat{g}(q)$ are estimation errors of the inertial matrix and the centrifugal and Coriolis torques respectively. This gives the system architecture for the closed loop system configuration as shown in Fig.3-1. The determination of u_a, however, requires the derivation of an adaptive algorithm, shown in the block "adaptive mechanism" in Fig.3-1, so that the tracking errors are as small as possible.

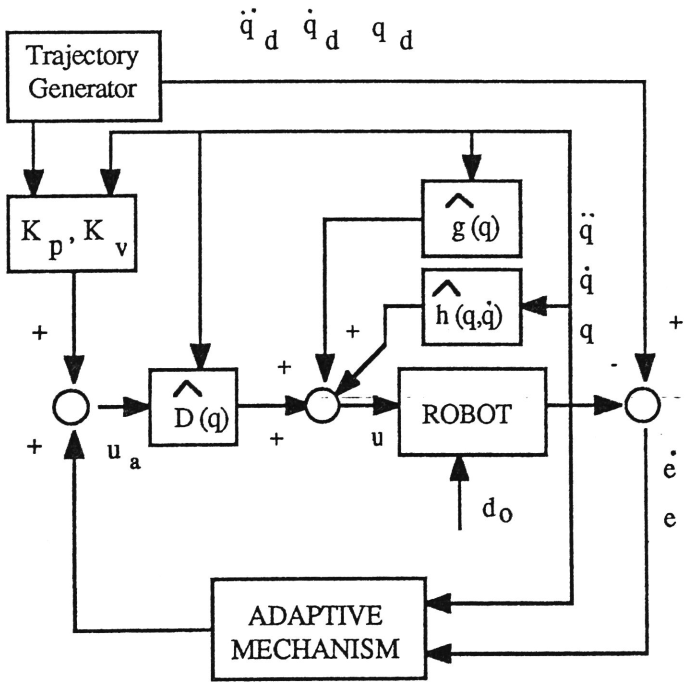

Fig.3-1. The overall system configuration.

In the case of pure computed torque control, i.e., $u_a=0$, Eqn.(3-3-2) will become

$$\ddot{e}+K_v\dot{e}+K_pe=\hat{D}^{-1}(q)[\tilde{D}(q)\ddot{q}+\tilde{h}(q,\dot{q})+\tilde{g}(q)+d_o].$$

The right hand side of the equation above has been shown by [13] to be a function of e, $\dot{e}$, and $\ddot{q}_d$:

$$\ddot{e}+K_v\dot{e}+K_pe=\xi$$

$$\xi=D^{-1}[\tilde{D}(q)\ddot{q}_d+\tilde{h}(q,\dot{q}_d)+\tilde{g}(q)+d_o+(I-D^{-1}\hat{D})K_pe$$

$$+(I-D^{-1}\hat{D})K_p\dot{e}-2\tilde{H}_m(q,\dot{q}_d)\dot{e}+\tilde{H}(q,\dot{e})].$$

Moreover, it has been proved, in [13], that the control law Eqn.(3-3-1b) leads to a L_∞ input-output stable system provided the following conditions are satisfied (for further details, see [13]):

(C-1) Assumption 3-3) holds;

(C-2) Assumption 2-2) holds;

(C-3) $k_{vi}^2=4k_{pi}>0$;

(C-4) d_o is bounded and its components are uncorrelated;

(C-5) $\beta_1\alpha_2+\beta_2\alpha_3+2\beta_2(\alpha_1\alpha_4)^{1/2}<1$, in which

$$\beta_1=1/k_p,\quad \beta_2=4\exp(-1)/k_v,$$

$$\alpha_1=\|D^{-1}(\tilde{D}(q)\ddot{q}_d+\tilde{h}(q,\dot{q}_d)+\tilde{g}(q)+d_o)\|_\infty,$$

$$\alpha_2=\|(I-D^{-1}\hat{D})K_p\|_{i\infty},$$

$$\alpha_3=\|(I-D^{-1}\hat{D})K_v-2D^{-1}\tilde{H}_m(q,\dot{q}_d)\|_{i\infty},$$

$$\alpha_4=\|D^{-1}\|_{i\infty}\max_i\|\tilde{h}_i(q)\|_{i\infty}.$$

(C-6) The initial conditions $e(0)=\dot{e}(0)=0$ are satisfied.

In (C-5), $\tilde{h}(q,\dot{q})=\tilde{h}(q,\dot{q}_d)-2\tilde{H}(q,\dot{q}_d)\dot{e}+\tilde{h}(q,\dot{e})$, where $\tilde{h}(q,\cdot)$ is obtained by replacing $\dot{q}$ by $(\cdot)$ in $\tilde{h}(q,\dot{q})$, and $\tilde{H}(q,\dot{q}_d)$ is the estimation error matrix of $H(q,\dot{q})$, in which $\dot{q}$ is replaced by $\dot{q}_d$. The L_∞ norms are defined in such a way that for a vector h, $\|h\|_\infty=\max_i \sup|h_i|$; for a matrix H, $\|H\|_{i\infty}=\max_i \sup \sum_j |h_{ij}|$.

Conditions (C-1) – (C-6) set up robustness properties in the sense of bounded-input bounded-output stability for the computed torque control law in the cases where there exist parameter errors and uncertainties. It should be stressed that since Conditions (C-1) and (C-2) identify Assumptions 3-3) and 2-2) respectively the additional conditions required, in addition to the assumptions made before, are (C-3) to (C-6).

Assumption 3-3), which requires a positive definite $\hat{D}(q)$, is not a particularly restrictive condition. If the non-linear functions of positions in every element of D(q) are all known (which only depend on the kinematics of the robot arms, e.g., whether the arm's joints are revolute or prismatic), it is always possible to choose the estimates of unknown parameters in such a way that they correspond to masses, inertia tensors and geometrical sizes or their combinations for a given robot. Since some values such as mass, and geometrical size are all positive, it is always possible to assign correct signs on the estimates of these true values or their combinations so that the resultant $\hat{D}(q)$ is positive definite. The only difference is that $\hat{D}(q)$ here may correspond to a different robot system (determined by the estimated masses, inertial tensors and sizes) instead of the robot defined by D(q), but $\hat{D}(q)$ must be positive definite because of its physical meaning. In order to explain this, consider the example in Section 2-2-3 again. In this example the estimate of the inertial matrix Eqn.(2-2-20), according to Eqn.(3-2-6a), is given by

$$\hat{D}(q)=\begin{bmatrix} \hat{d}_{11}(q) & \hat{d}_{12}(q) \\ \hat{d}_{21}(q) & \hat{d}_{22}(q) \end{bmatrix}$$

$$=\begin{bmatrix} \hat{c}_1+\hat{c}_3+\hat{c}_4\hat{m}_L+(\hat{c}_2+2\hat{L}_1\hat{L}_2\hat{m}_L)\cos q_2 & \hat{c}_3+\hat{L}_2^2\hat{m}_L+(\hat{c}_2/2+\hat{L}_1\hat{L}_2\hat{m}_L)\cos q_2 \\ \hat{c}_3+\hat{L}_2^2\hat{m}_L+(\hat{c}_2/2+\hat{L}_1\hat{L}_2\hat{m}_L)\cos q_2 & \hat{c}_3+\hat{L}_2^2\hat{m}_L \end{bmatrix}$$

(3-3-3)

where

$$\hat{c}_1 = \hat{L}_1^2(\hat{m}_1/3+\hat{m}_2)$$

$$\hat{c}_2 = \hat{L}_1\hat{L}_2\hat{m}_2$$

$$\hat{c}_3 = \hat{L}_2^2\hat{m}_2/3$$

$$\hat{c}_4 = \hat{L}_1^2+\hat{L}_2^2.$$

Clearly, the positive definiteness of Eqn.(3-3-3) can be proven if:

$$\hat{d}_{11}(q)=\hat{c}_1+\hat{c}_3+\hat{c}_4\hat{m}_L+(\hat{c}_2+2\hat{L}_1\hat{L}_2\hat{m}_L)\cos q_2 >0 \qquad (3\text{-}3\text{-}4)$$

and

$$\det \hat{D}(q)>0, \qquad (3\text{-}3\text{-}5)$$

where det $\hat{D}(q)$ expresses the determinant of matrix $\hat{D}(q)$.

As in Section 2-2-3, it can be shown that for any $\hat{L}_1>0$, $\hat{L}_2>0$, $\hat{m}_1>0$, $\hat{m}_2>0$ and $\hat{m}_L>0$, Eqn.(3-3-4) and (3-3-5) hold, as

$$\hat{c}_1+\hat{c}_3+\hat{c}_4\hat{m}_L+(\hat{c}_2+2\hat{L}_1\hat{L}_2\hat{m}_L)\cos q_2$$

$$> \hat{c}_1+\hat{c}_3+\hat{c}_4\hat{m}_L-(\hat{c}_2+2\hat{L}_1\hat{L}_2\hat{m}_L)$$

$$=(\hat{L}_1\sqrt{\hat{m}_1/3+\hat{m}_2}-\hat{L}_2\sqrt{\hat{m}_2/3})^2+(\hat{L}_1-\hat{L}_2)^2\hat{m}_L+2\hat{L}_1\hat{L}_2\sqrt{(\hat{m}_1/3+\hat{m}_2)\hat{m}_2/3}-\hat{L}_1\hat{L}_2\hat{m}_2$$

$$>0$$

and

$$\det \hat{D}(q) > \hat{L}_1^2 \hat{L}_2^2(\hat{m}_1\hat{m}_2/9+\hat{m}_2^2/12+\hat{m}_2\hat{m}_L/3+\hat{m}_1\hat{m}_L/3)>0$$

This means, in this example, the only condition needed to ensure a positive definite estimate of D(q) is that all estimates of dynamic parameters L_i, m_i and m_L are greater than zero.

It should be emphasised again that the estimated parameters in non-adaptive control law Eqn.(3-3-1b) are fixed by a set of a priori values and the adaptive control is realized by control component u_a in Eqn.(3-3-1c). Compared with the adaptive control approaches given in [12] and [43], in which $\hat{D}(q)$ is updated by the estimator, the advantage of the approach presented here is that the positive definiteness of $\hat{D}(q)$ can be guaranteed at all times and it is not necessary to project the estimated parameters into a certain range in parameter space or to "cut off" the estimation when the estimated parameters exceed certain boundaries to ensure the positive definiteness of $\hat{D}(q)$ as required by other approaches.

3-3-2. Decentralized Error System Structure

The error dynamic system Eqn.(3-3-2) is a set of simultaneous equations coupled by the inverse of the estimated inertial matrix $\hat{D}^{-1}(q)$ together with system states q and $\dot{q}$. The exact decoupling for these subsystems is still an open question. In this section, a novel structure will be proposed to show that a partial decoupling can be achieved by means of splitting $\hat{D}^{-1}(q)$ into two parts. The first part is a diagonal matrix consisting of all diagonal elements of $\hat{D}^{-1}(q)$. The second part is composed of all non-diagonal elements of $\hat{D}^{-1}(q)$ with all diagonal elements replaced by zeros. With this treatment, Eqn.(3-3-2) can be considered as n separate subsystems (affected by the first part of $\hat{D}^{-1}(q)$) with interconnections between them (affected by the second part of $\hat{D}^{-1}(q)$). For this system architecture it will be shown in Chapters 4 ~~and~~ 5 that robust controllers can be designed for each error subsystem to ensure bounded position and velocity tracking errors.

Denote

$$\hat{D}^{-1}(q)=\hat{J}(q)=\hat{J}_d(q)+\hat{J}_o(q),$$

where

$$\hat{J}_d(q)=\mathrm{diag}\{[\hat{D}^{-1}(q)]_{ii}\}=\mathrm{diag}\{\hat{J}_{ii}(q)\} \tag{3-3-6}$$

is a diagonal matrix consisting of all diagonal elements of $\hat{D}^{-1}(q)$, and $\hat{J}_o(q)$ is a matrix obtained by replacing all diagonal elements of $\hat{D}^{-1}(q)$ by zeros. Then equation Eqn.(3-3-2) becomes

$$\ddot{e}+K_v\dot{e}+K_pe=\hat{J}_d(q)[\bar{D}(q)\ddot{q}+\bar{h}(q,\dot{q})+\bar{g}(q)]-u_a+d, \tag{3-3-7}$$

where

$$d=\hat{J}_o(q)[\bar{D}(q)\ddot{q}+\bar{h}(q,\dot{q})+\bar{g}(q)]+\hat{D}^{-1}(q)d_o\in R^n. \tag{3-3-8}$$

As mentioned in Section 3-3-1, if conditions (C1) – (C-6) are satisfied, control law Eqn.(3-3-1b) ensures input-output stability and the error state will stay within a bounded region including the origin. It has also been shown by [13], that in the case $u_a=0$, the right hand side of Eqn.(3-3-2) is bounded, which implies, $\hat{J}_o(q)(\bar{D}(q)\ddot{q}+\bar{h}(q,\dot{q})+\bar{g}(q))$ in Eqn.(3-3-7) is bounded as well. Suppose that this bound is given by a constant v>0, then

$$\|d(t)\| \le \|\hat{J}_o(q)(\bar{D}(q)\ddot{q}+\bar{h}(q,\dot{q})+\bar{g}(q))\|+\|\hat{D}^{-1}(q)d_o\| \le v+\|\hat{J}(q)d_o\| \le \rho_o$$

since $\hat{D}^{-1}(q)$ is bounded, where $\rho_o>0$ is a constant.

For the first two terms inside the brackets in the right hand side of Eqn.(3-3-7),

$$\bar{D}(q)\ddot{q}+\bar{h}(q,\dot{q})=\bar{D}(q)\ddot{q}+h(q,\dot{q})-\hat{h}(q,\dot{q}).$$

Recalling Eqn.(3-2-2a) and Assumption 3-4), the equation above can be rewritten as

$$\bar{D}(q)\ddot{q}+\bar{h}(q,\dot{q})=\bar{D}(q)\ddot{q}+\dot{D}(q)\dot{q}+s(q,\dot{q})-\dot{\hat{D}}(q)\dot{q}-\hat{s}(q,\dot{q})$$

$$=\bar{D}(q)\ddot{q}+\dot{\bar{D}}(q)\dot{q}+\bar{s}(q,\dot{q})$$

$$=\frac{d}{dt}(\bar{D}(q)\dot{q})+\bar{s}(q,\dot{q}). \tag{3-3-9}$$

Thus, Eqn.(3-3-7) becomes

$$\ddot{e}+K_v\dot{e}+K_pe=\hat{J}_d(q)[\frac{d}{dt}(\bar{D}(q)\dot{q})+\bar{s}(q,\dot{q})+\bar{g}(q)]-u_a+d. \tag{3-3-10}$$

The Eqn.(3-3-10) can be considered as n separate multi-input-single-output subsystems. Then the i-th subsystem becomes

$$\ddot{e}_i+k_{vi}\dot{e}_i+k_{pi}e_i=\hat{J}_{ii}(q)[\frac{d}{dt}(\bar{D}_i(q)\dot{q})+\bar{s}_i(q,\dot{q})+\bar{g}_i(q)]-u_{ai}+d_i, \tag{3-3-11}$$

where $\hat{J}_{ii}(q)$ is the i-th diagonal element of $\hat{J}_d(q)$ (see Eqn.(3-3-6)), $\bar{D}_i(q)$ is the i-th row of matrix $\bar{D}(q)$ and d_i is the i-th component of d.

According to Eqn.(3-3-8) and (3-3-9), d_i in Eqn.(3-3-11) can be rewritten as

$$\begin{aligned}d_i &= d_i(q,\dot{q},\ddot{q})\\ &=\sum_{j=1 j\neq i}^{n}[\hat{J}_{ij}(q)(\sum_{k=1}^{n}\frac{d}{dt}(\bar{D}_{jk}(q)\dot{q}_k)+\bar{s}_j(q,\dot{q})+\bar{g}_j(q))]+\sum_{j=1}^{n}\hat{J}_{ij}(q)d_{oj}\\ &=\sum_{j=1 j\neq i}^{n}\sum_{k=1}^{n}\hat{J}_{ij}(q)\frac{d}{dt}(\bar{D}_{jk}(q)\dot{q}_k)+\sum_{j=1 j\neq i}^{n}\hat{J}_{ij}(q)\bar{s}_j(q,\dot{q})+\sum_{j=1 j\neq i}^{n}\hat{J}_{ij}(q)\bar{g}_j(q)+\sum_{j=1}^{n}\hat{J}_{ij}(q)d_{oj},\end{aligned} \tag{3-3-12}$$

where $\bar{D}_{ij}(q)$ is the i-j-th element of the matrix $\bar{D}(q)$ and $\hat{J}_{ij}(q)$ the i-j-th element of $\hat{J}(q)=\hat{D}^{-1}(q)$.

Since d is bounded, d_i must be bounded as well. It is supposed that this boundedness is given by a constant ρ_{oi}, i.e.,

$$|d_i|<\rho_{oi}. \tag{3-3-13}$$

In accordance with Assumption 3-1), known nonlinear functions and unknown estimation errors in the equalities above can be decomposed as two vectors so that Eqn.(3-3-11) becomes

$$\ddot{e}_i+k_{vi}\dot{e}_i+k_{pi}e_i=\hat{J}_{ii}(q)[\frac{d}{dt}(\omega_{i1}^T(q,\dot{q}))\bar{\theta}_{i1}+\omega_{i2}^T(q,\dot{q})\bar{\theta}_{i2}+\omega_{i3}^T(q)\bar{\theta}_{i3}]-u_{ai}+d_i \tag{3-3-14a}$$

$$=\bar{\theta}_i^{T}\hat{J}_{ii}(q)\omega_i(q,\dot{q},\ddot{q})-u_{ai}+d_i, \tag{3-3-14b}$$

where

$$\omega_i^T = [\frac{d}{dt}(\omega_{i1}^T(q,\dot{q})) \quad \omega_{i2}^T(q,\dot{q}) \quad \omega_{i3}^T(q)],$$

$$\bar{\theta}_i^T = [\ \bar{\theta}_{i1}{}^T \quad \bar{\theta}_{i2}{}^T \quad \bar{\theta}_{i3}{}^T\ ],$$

$$\omega_{i1}^T = [f_{i11}(q)\dot{q}_1,..,f_{i1m_i}(q)\dot{q}_1,f_{i21}(q)\dot{q}_2,..,f_{i2m_i}(q)\dot{q}_2,..,f_{in1}(q)\dot{q}_n,..,f_{inm_i}(q)\dot{q}_n],$$

$$\omega_{i2}^T = [f_{ki1}(q,\dot{q}),\ f_{ki2}(q,\dot{q}),\dots,\ f_{kim_{ik}}(q,\dot{q})],$$

$$\omega_{i3}^T = [f_{gi1}(q),\ f_{gi2}(q),\dots,\ f_{gim_{ig}}(q)],$$

$$\bar{\theta}_{i1}^T = [\bar{d}_{i11},\dots,\bar{d}_{i1m_i},\bar{d}_{i21},\dots,\bar{d}_{i2m_i},\dots,\bar{d}_{in1},\dots,\bar{d}_{inm_i}],$$

$$\bar{\theta}_{i2}^T = [\bar{s}_{i1},\bar{s}_{i2},\dots,\bar{s}_{imik}],$$

$$\bar{\theta}_{i3}^T = [\bar{g}_{i1},\ \bar{g}_{i2},\ \dots,\ \bar{g}_{imig}].$$

Being a constant vector, $\bar{\theta}_i$ is the parameter estimation error caused by the computed torque control law Eqn.(3-3-1b).

Eqn.(3-3-14b) is the resultant error subsystem structure. In Chapters 4 and 5, it will be shown how the adaptive control law for each subsystem can be determined. It is worth noting that using this decentralized system structure in the right hand side of each subsystem, as shown by Eqn.(3-3-14b), the unknown parameter error $\bar{\theta}_i$ and measureable state function $\hat{J}_{ii}(q)\omega_i(q,\dot{q},\ddot{q})$ are both vectors and therefore they are commutative, i.e., $\bar{\theta}_i^T\hat{J}_{ii}(q)\omega_i(q,\dot{q},\ddot{q})=(\hat{J}_{ii}(q)\omega_i(q,\dot{q},\ddot{q}))^T\bar{\theta}_i$. This is not true for the multi-variable system structure given by Eqn.(2-4-3) where $\hat{D}^{-1}(q)\Omega(q,\dot{q},\ddot{q})$ and $\bar{\theta}$ cannot be interchanged as $\hat{D}^{-1}(q)\Omega(q,\dot{q},\ddot{q})$ is a matrix. This commutative property of Eqn.(3-3-14b) is important for adaptive controller design. As will be shown in Chapter 5, it is possible to introduce a linear operator to act on $\bar{\theta}_i$ so that acceleration measurement can be avoided and an additional zero can be introduced into the error system to satisfy the postive real condition.

3-3-3. Comments

In system Eqn.(3-3-11), the first term $\hat{J}_{ii}(q)[\frac{d}{dt}(\bar{D}_i(q)\dot{q})+\bar{s}_i(q,\dot{q})+\bar{g}_i(q)]$ is considered as a dominant input for the subsystem i and has been parameterized as $\bar{\theta}_i^T\hat{J}_{ii}(q)\omega_i(q,\dot{q},\ddot{q})$ in Eqn.(3-3-14b). The other terms which are the functions of $q,\dot{q},\ddot{q}$ and unknown constant parameter estimate errors, are combined in term d_i representing interconnections among different subsystems and uncertainties. However, if $\bar{\theta}_i=0$, which means that the parameters employed in the computed torque are all the true values of the system dynamic parameters, and $d_{oi}=0$, which means there are no structural uncertainties, then the right hand side of Eqn.(3-3-14b) will disappear and u_{ai} will no longer necessarily exist. In the cases where there are parameter errors in the computed torque control law, the design objective is to derive an adaptive control law to compensate the dominant term so that the tracking error e_i becomes as small as possible.

It also can be seen, from Eqn.(3-3-11) and Eqn.(3-3-14b), that even if d_i is ignored each subsystem is still a multi-input single-output system in which the term $\tilde{\theta}_i^T \hat{J}_{ii}(q)\omega_i(q,\dot{q},\ddot{q})$ is a scalar function of q, $\dot{q}$ and $\ddot{q}$ and dynamic parameter errors and can be regarded as the dominant input . This means that the subsystem i is not insulated from others completely and is still coupled with them by q_j, $\dot{q}_j$ and $\ddot{q}_j$ ($j=1,2,\ldots,n,\ j\neq i$) through $\hat{J}_{ii}(q)\omega_i(q,\dot{q},\ddot{q})$. In order to compensate the dominant term, it is necessary for the adaptive control to utilize information about other joints' positions, velocities and accelerations as will be shown in the following chapters. This is therefore not a pure local feedback decentralized system in which the dominant input to subsystem i is only the function of the state of subsystem i. However, as robot systems are more compact than many decentralized systems, the information required can be easily exchanged between the microprocessors controlling the different joints of the robot arms.

3-4. SUMMARY

In this chapter, a two component controller structure is proposed for the dynamic equation of a robot system. As the result of the first (non-adaptive) control component, an error dynamic equation is obtained. In order to avoid the computation of the inverse of the estimated inertia matrix, the overall error dynamics is split into n subsystems by considering the terms which are related to the diagonal elements of the inverse of the inertia matrix as the dominant input to the error equations. Meanwhile the other non-dominant terms are taken into account using interconnections between subsystems. In Section 3-3, it has been shown that the disturbances d_i are bounded under condition (C-1) to (C-6) given in Section 3-2. This is very important in the design of adaptive components as will be shown in Chapter 4. Another advantage of this structure is that since each resultant subsystem is a scalar system it will provide more flexibility in adaptive controller design since in this case the unknown parameter and measurement of the system state are both vectors so that they are commutative.

Chapter 4.

ROBUST ADAPTIVE CONTROLLER

4-1. INTRODUCTION

In this chapter a novel adaptive control approach based on the decentralized error system structure obtained in Chapter 3 will be investigated [40]. The main result is a novel adaptive control algorithm. Since uncertainties in the system modelling are taken into account and the interactions among the subsystems are regarded as disturbances, the resulting controller is robust in this environment. A proof of stability and analytical results of the boundedness of position tracking errors will be given. By introducing linear operators in state measurements the approach also avoids the difficulty of measuring the accelerations of the joints of the robot arms.

Utilizing Property 2-3), the robot dynamic equations can be written to be linear in model parameters such as masses, inertia tensors and payload of the robot arms. This makes it possible to employ linear adaptive control techniques in robot system adaptive controller design provided positions and velocities are measurable and the nonlinear functions are all known. There has also been further research on the stability and convergence of these techniques. Two types of approach have been proposed using these "linear" descriptions. One is based on model following [12] and [13] and the other on the passivity of robot dynamics [55]–[58].

In the case of model following, Craig in [12] and [13] proposed an adaptive control method based on the computed torque control law. This approach leads to an asymptotically globally stable closed-loop system in the sense of Lyapunov stability. An outline of this method was introduced in Section 2-4. However, this method has three drawbacks: it requires the whole inverse of the estimated inertia matrix to be calculated, the parameter estimate must always lie within certain ranges so that the estimated inertia matrix is non-singular, and the joint accelerations of the robotarms must be measured. The investigations given in this chapter are aimed at relaxing these restrictions and obtaining a robust control algorithm in the cases where there exist uncertainties in the modelling and the environment.

In Chapter 3, it has been shown that based on the a priori parameter estimates of the system equations, a computed torque control can be applied so that the controlled states are driven near to the desired trajectories. Then the resultant error dynamics can be treated as a set of multi-input single-output error systems, in which the unknown parameters appear in a form linear in the generalized states, and interactions between the different subsystems are regarded as disturbances. For this decentralized error system configuration a robust adaptive control component will be presented in this chapter using the Lyapunov direct method [28][34].

The adaptive controller design is presented in Section 4-2. In Section 4-2-1, a linear operator is introduced. As a result of manipulating the subsystem stucture, given in Chapter 3, by this operator, a new filtered error dynamics can be obtained. It can be shown that for this filtered error subsystem, the direct measurements of the joint accelerations of robot arms can be avoided with the interconnections still bounded. In Section 4-2-2, the structure of the adaptive control component is introduced and the adaptive control law is presented. In Section 4-2-3, the stability of the error system will be investigated. It will be shown that the filtered tracking position errors are bounded in a residual set which is proportional to the bounds of the interconnections. Furthermore, a corollary is presented to show the boundedness of the real position tracking errors.

In Section 4-3, some comments on the controller will by given and a summary is provided in Section 4-4.

4-2. ROBUST ADAPTIVE CONTROLLER DESIGN

Subsystem i, given by Eqn.(3-3-14b), appears as a standard linear system with a disturbance term d_i and a compensating control term u_{ai} which is intended to be a robust adaptive control law. In this section, a linear operator which is used in state measurement will be introduced. Then the design of u_{ai} and the stability analysis on the resulting closed-loop system will be presented.

4-2-1. Linear Operator

As $\omega_i(q,\dot{q},\ddot{q})$ in Eqn.(3-3-14b) is a function of the accelerations $\ddot{q}$, direct adaptive controller design based on Eqn.(3-3-14b) may require the measurement of accelerations which is technically difficult in practice. In order to avoid this, a linear operator is introduced into both sides of Eqn.(3-3-14b).

The linear operator is given by

$$L(s)=s+\alpha, \tag{4-2-1}$$

where $\alpha>0$ is a constant so that L(s) is a Hurwitz polynomial and "s" the differential operator specified by $s(\cdot)=d(\cdot)/dt$. The inverse operator of L(s) is

$$L^{-1}(s)=\frac{1}{s+\alpha}. \tag{4-2-2}$$

It is easy to show that the operators L(s) and $L_i^{-1}(s)$ are linear, i.e., for instance

$$L(s)(c_1f_1(t))=c_1L(s)f_1(t), \tag{4-2-3}$$

and

$$L(s)(c_1f_1(t)+c_2f_2(t))=c_1L(s)f_1(t)+c_2L(s)f_2(t). \tag{4-2-4}$$

where $f_1(t)$ and $f_2(t)$ are functions of time, and c_1 and c_2 are all real numbers.

In order to avoid measuring $\ddot{q}$, both sides of subsystem Eqn.(3-3-14b) are fed into the inverse operator

$$L_i^{-1}(s)=\frac{1}{s+\alpha_i}, \tag{4-2-5}$$

where $\alpha_i>0$ is a constant and the operator can also be regarded as a filter.

By this treatment, and denoting

$$\varepsilon_i = \frac{1}{s+\alpha_i} e_i, \tag{4-2-6a}$$

$$\bar{\theta}_i^T \delta_i(q,\dot{q}) = \frac{1}{s+\alpha_i}[\bar{\theta}_i^T \hat{J}_{ii}(q)\omega_i(q,\dot{q},\ddot{q})]= \bar{\theta}_i^T \frac{1}{s+\alpha_i}[\hat{J}_{ii}(q)\omega_i(q,\dot{q},\ddot{q})] \tag{4-2-6b}$$

$$\eta_i = \frac{1}{s+\alpha_i} d_i, \tag{4-2-6c}$$

$$\tau_{ai} = \frac{1}{s+\alpha_i} u_{ai}, \tag{4-2-6d}$$

Eqn.(3-3-14b) becomes

$$\ddot{\varepsilon}_i+k_{vi}\dot{\varepsilon}_i+k_{pi}\varepsilon_i= \bar{\theta}_i^T\delta_i(q,\dot{q})+\eta_i-\tau_{ai}, \tag{4-2-7}$$

where $\bar{\theta}_i$=constant is the estimated parameter vector with a dimension of $z_i=nm_i+m_{ik}+m_{ig}$, and $\delta_i(q,\dot{q})$ is the filtered observation vector formed by a set of known nonlinear functions of the states.

Eqn.(4-2-6b) arises from the linear property of the inverse operator $L_i^{-1}(s)$ and $\delta_i(q,\dot{q})$ is given by

$$\delta_i^T(q,\dot{q}) = [\ \delta_{i1}^T(q,\dot{q}),\ \delta_{i2}^T(q,\dot{q}),\ \delta_{i3}^T(q)\ ]$$

$$= \frac{1}{s+\alpha_i}\ [\hat{J}_{ii}(q)\ \frac{d}{dt}(\omega_{i1}^T(q,\dot{q})),\ \hat{J}_{ii}(q)\omega_{i2}^T(q,\dot{q}),\ \hat{J}_{ii}(q)\omega_{i3}^T(q)]$$

$$=[\ \frac{1}{s+\alpha_i}\ \hat{J}_{ii}(q)\frac{d}{dt}\omega_{i1}^T(q,\dot{q}),\ \frac{1}{s+\alpha_i}\hat{J}_{ii}(q)\omega_{i2}^T(q,\dot{q}),\ \frac{1}{s+\alpha_i}\hat{J}_{ii}(q)\omega_{i3}^T(q)]. \tag{4-2-8}$$

It is easy to see that there is no need to measure the accelerations in the implementations of the second and third components of the right hand side of Eqn.(4-2-8). The following will show that this is also true for the implementation of the first component. In view of Eqn.(4-2-8), $\delta_{i1}(q,\dot{q})$ can be regarded as the output of the filter operator to which $\hat{J}_{ii}(q)\frac{d}{dt}\omega_{i1}(q,\dot{q})$ is the input, i.e.,

$$\delta_{i1}(q,\dot{q})=\frac{1}{s+\alpha_i}\ \hat{J}_{ii}(q)\frac{d}{dt}\omega_{i1}(q,\dot{q}).$$

It is equivalent to a first order differential equation:

$$\dot{\delta}_{i1}(q,\dot{q})+\alpha_i\delta_{i1}(q,\dot{q})=\hat{J}_{ii}(q)\frac{d}{dt}\omega_{i1}(q,\dot{q}).$$

The solution of it is

$$\delta_{i1}(q,\dot{q})=e^{-\alpha_i t}\delta_{i1}(q(0),\dot{q}(0))+\int_0^t e^{-\alpha_i(t-\tau)}\ \hat{J}_{ii}(q(\tau))\ d\omega_{i1}(q(\tau),\dot{q}(\tau)).$$

The integral term in the right hand side above can be solved by means of integrating by parts, i.e., (see [43])

$$\int_0^t e^{-\alpha_i(t-\tau)}\ \hat{J}_{ii}(q(\tau))d\omega_{i1}(q(\tau),\dot{q}(\tau))$$

$$=e^{-\alpha_i(t-\tau)}\ \hat{J}_{ii}(q(\tau))\omega_{i1}(q(\tau),\dot{q}(\tau))\Big|_0^t-\int_0^t \omega_{i1}(q(\tau),\dot{q}(\tau))d(e^{-\alpha_i(t-\tau)}\ \hat{J}_{ii}(q(\tau)))$$

which, obviously, excludes the accelerations.

Since d_i is bounded by ρ_{oi}, according to Assumption 3-6), $\eta_i(q)$ in Eqn.(4-2-6c) will be bounded by a constant $\rho_i=|\eta_i(0)|+\rho_{oi}/\alpha_i$, i.e.,

$$|\eta_i(t)|\le|\eta_i(0)|+\rho_{oi}/\alpha_i=\rho_i. \qquad (4\text{-}2\text{-}9)$$

This can be shown by considering the solution of the differential equation defined by Eqn.(4-2-6c)

$$\dot{\eta}_i+\alpha_i\eta_i=d_i.$$

As d_i is a function of time, the solution is

$$\eta_i(t)=e^{-\alpha_i t}\eta_i(0)+\int_0^t e^{-\alpha_i(t-\tau)}d_i(\tau)d\tau.$$

Because d_i is bounded by ρ_{oi}, the boundedness of $\eta_i(t)$ can be shown by

$$\begin{aligned}|\eta_i(t)| &\le e^{-\alpha_i t}|\eta_i(0)|+\int_0^t e^{-\alpha_i(t-\tau)}|d_i(\tau)|d\tau \\ &\le |\eta_i(0)|+\int_0^t e^{-\alpha_i(t-\tau)}\rho_{oi}d\tau \\ &= |\eta_i(0)|+\frac{1}{\alpha_i}(1-e^{-\alpha_i t})\rho_{oi} \\ &\le |\eta_i(0)|+\rho_{oi}/\alpha_i = \rho_i,\end{aligned}$$

which is Eqn.(4-2-9).

4-2-2. Adaptive Control Algorithm

For error subsystem Eqn.(4-2-7), this section will be presenting the design of the adaptive control component τ_{ai}. Suppose that τ_{ai} has the form of

$$\tau_{ai}=\hat{\theta}_i^T\delta_i(q,\dot{q}), \tag{4-2-10}$$

where $\hat{\theta}_i\in R^{z_i}$ is an estimate of θ_i. Substituting Eqn.(4-2-10) into Eqn.(4-2-7) gives

$$\begin{aligned}\ddot{\varepsilon}_i+k_{vi}\dot{\varepsilon}_i+k_{pi}\varepsilon_i &=(\theta_i-\hat{\theta}_i)^T\delta_i(q,\dot{q})+\eta_i \\ &= \phi_i^T\delta_i(q,\dot{q})+\eta_i,\end{aligned} \tag{4-2-11}$$

where $\phi_i=\theta_i-\hat{\theta}_i$. Define a state vector $x_i\in R^2$:

$$x_i = \begin{bmatrix} \varepsilon_i \\ \dot{\varepsilon}_i \end{bmatrix} \tag{4-2-12}$$

then subsystem i can be expressed by state space description

$$\dot{x}_i = A_i x_i + b_i \phi_i^T \delta_i(q,\dot{q}) + b_i \eta_i, \tag{4-2-13}$$

where

$$A_i = \begin{bmatrix} 0 & 1 \\ -k_{pi} & -k_{vi} \end{bmatrix} \qquad b_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{4-2-14}$$

As A_i is a stable matrix, for a given positive definite $Q_i = Q_i^T > 0$, there exists a positive definite matrix $P_i^T = P_i > 0$ such that the Lyapunov equation

$$A_i^T P_i + P_i A_i = -Q_i \tag{4-2-15}$$

is satisfied, for i=1,2,...,n.

Error system structure Eqn.(4-2-11) or Eqn.(4-2-13) appears as a subsystem structure which is similar to that given in many publications (see, e.g., [27], [18] and [53]). The only difference is that in Eqn.(4-2-13) the term $b_i \phi_i^T \delta_i(q,\dot{q})$ is a function of global states q, $\dot{q}$ rather than the local state q_i and $\dot{q}_i$. It is possible to separate $b_i \phi_i^T \delta_i(q,\dot{q})$ into two parts which depend on local and global states separately so that the former can be attributed to the interconnections and the adaptive control component is used to compensate the latter only. However, in order to obtain good performance the controller design is based on Eqn.(4-2-13) in which the global feedback structure is applied.

As a result of this, the overall system then becomes an error state equation with dimension 2n:

$$\dot{x} = Ax + B\Phi\delta + B\eta, \tag{4-2-16}$$

where

$$A=\mathrm{diag}\{A_1\ A_2 \ldots A_n\} \in R^{2nx2n}$$

$$B=\mathrm{diag}\{b_1\ b_2 \ldots b_n\} \in R^{2nxn}$$

$$\Phi^T=\mathrm{diag}\{\phi_1^T\ \ \phi_2^T \ldots \phi_3^T\} \in R^{nx\Delta}$$

$$\delta=[\delta_1^T\ \delta_2^T \ldots \delta_3^T] \in R^{\Delta}$$

and

$$\eta^T=[\eta_1^T\ \ \eta_2^T \ldots \eta_3^T] \in R^n$$

where $\Delta = \sum_{i=1}^{n} z_i$.

For error subsystem Eqn.(4-2-13), the following robust control strategy is introduced

$$\dot{\phi}_i = - \beta_i \phi_i - \gamma_i\, b_i^T P_i x_i \delta_i \qquad \text{for } i=1,2,\ldots,n. \qquad (4\text{-}2\text{-}17)$$

where β_i, $\gamma_i>0$ are constants, $P_i^T=P_i>0$ are the solution of the Lyapunov equation Eqn.(4-2-15).

The i-th error subsystem is shown in Fig.4-1.

4-2-3. Stability and Convergence

In Section 4-2-2, the error system Eqn.(4-2-13) and a robust controller Eqn.(4-2-17) have been derived. In this section some theoretical results on the stability of the closed loop system will be stated. The main result is Theorem 4-1, which shows the quantitative boundedness of the error state of Eqn.(4-2-13). According to this theorem, the bounded position tracking error can also be derived which will be given by a Corollary of Theorem 4-1. Moreover a geometrical interpretation of the theorem will be given.

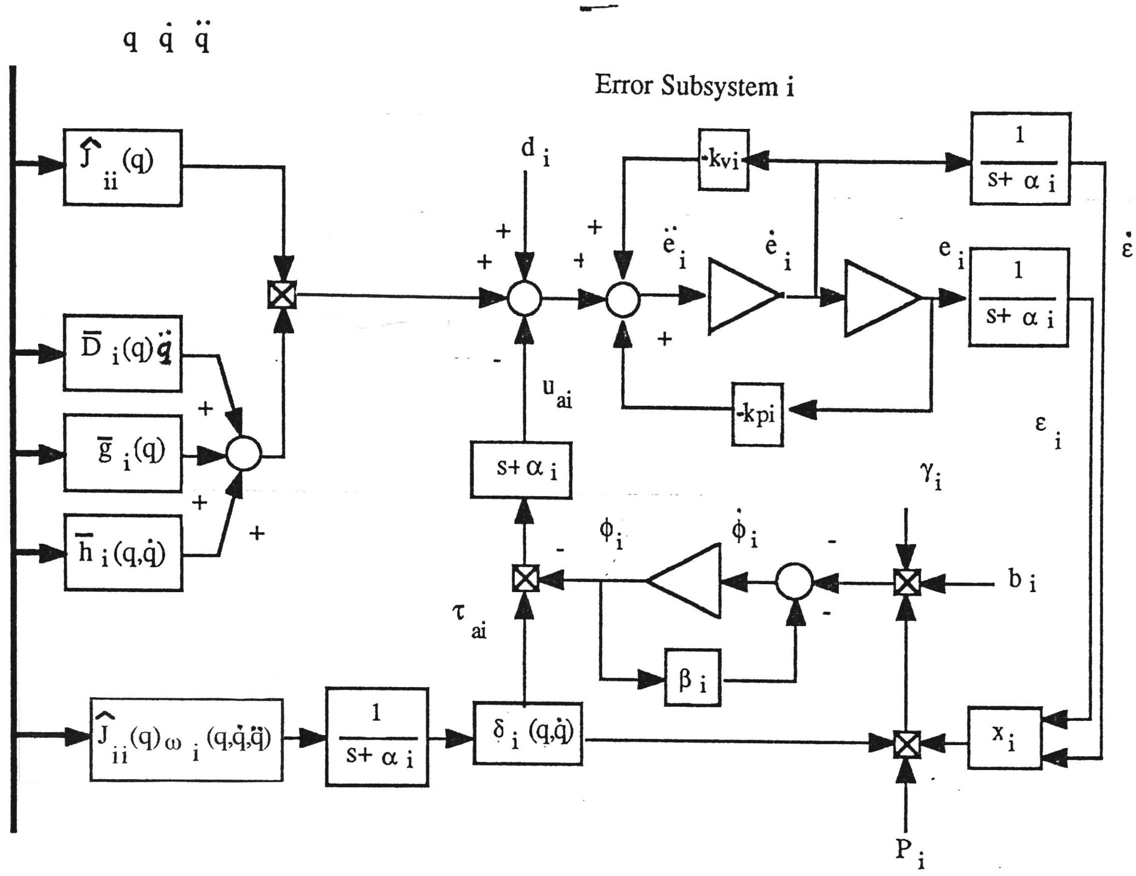

Fig.4-1. The architecture of error subsystem i.

The following theorem gives the main result on the stability of the system Eqn.(4-2-13) and the controller Eqn.(4-2-17):

Theorem 4-1:

Using the control law Eqns.(3-3-1a,b, and c) together with Eqn.(4-2-6d) and Eqn.(4-2-10) in which $\hat{\theta}_i=\theta_i-\phi_i$, and Eqn.(4-2-17), provided all assumptions are true as well as conditions (C-3)-(C-6) are satisfied for Eqn.(3-3-1b), then:

(i) The solutions x_i and ϕ_i of the i-th error equation Eqn.(4-2-13) and adaptive controller Eqn.(4-2-17) are uniformly bounded;

(ii) There exist constants $\upsilon_i>0$, for the adaptive control parameter $\beta_i>0$ in Eqn.(4-2-16), for which x_i and ϕ_i will converge to the residual set

$$D_i=\{(x_i, \phi_i)|v_i(x_i, \phi_i)<\frac{1}{2\upsilon_i\sigma_i} \max\lambda^2(P_i)\rho_i^2\} \tag{4-2-18}$$

with a rate at least as fast as $\exp(-\upsilon_i t)$, where ρ_i, given by Eqn.(4-2-9), is the super bound of the uncertainties in subsystem i, and $\sigma_i=\frac{1}{2}\min\lambda(Q_i)$ with $\min\lambda(Q_i)$ being the minimum eigenvalue of Q_i given by the Lyapunov equation Eqn.(4-2-18).

(iii) Furthermore, according to (i) and (ii), the solution x and Φ of the overall system Eqn.(4-2-16) will converge to the residual set

$$D_o=\{(x,\Phi)|v(x,\Phi)<\frac{1}{\min\upsilon_i} \sum_{i=1}^{n}\frac{1}{2\sigma_i} \max\lambda^2(P_i)\rho_i^2\} \tag{4-2-19}$$

with a rate at least as fast as $\exp(-\min\upsilon_i t)$.

Proof:

The proof is similar to the method given in [25]and [36]. Consider a candidate for the Lyapunov function for the i-th subsystem Eqn.(4-2-7):

$$v_i(x_i,\phi_i) = \frac{1}{2} x_i^T P_i x_i + \frac{1}{2\gamma_i} \phi_i^T\phi_i. \tag{4-2-20}$$

Its total derivative along the solution trajectory of Eqn.(4-2-7) is

$$\dot{v}_i(x_i,\phi_i)= \frac{1}{2}x_i^T(A_i^TP_i+P_iA_i)x_i+ x_i^T P_ib_i\phi_i^T\delta_i + \frac{1}{\gamma_i}\phi_i^T\dot{\phi}_i + x_i^TP_ib_i\eta_i \tag{4-2-21}$$

Applying Eqn.(4-2-17) and Eqn.(4-2-15) leads to

$$\dot{v}_i(x_i,\phi_i)= - \frac{1}{2} x_i^TQ_ix_i + x_i^TP_ib_i\delta_i^T\phi_i + \frac{1}{\gamma_i}\phi_i^T(- \beta_i\phi_i - \gamma_ib_i^T P_ix_i\delta_i)+ x_i^TP_ib_i\eta_i$$

$$\leq - \frac{1}{2} \min\lambda(Q_i)\|x_i\|^2 - \frac{1}{\gamma_i} \beta_i\|\phi_i\|^2 + \max\lambda(P_i)\|b_i\| \|x_i\| |\eta_i|$$

Denoting

$$\sigma_i = \frac{1}{2}\min\lambda(Q_i),$$

and noting that $\|b_i\| = \|[0,1]^T\| = 1$, then

$$\dot{v}_i(x_i,\phi_i) \le -\sigma_i \|x_i\|^2 - \frac{1}{\gamma_i}\beta_i\|\phi_i\|^2 + \max\lambda(P_i)\,\|x_i\|\,|\eta_i|$$

By completing the squares

$$\dot{v}_i(x_i,\phi_i) \le -\frac{1}{2}\sigma_i\|x_i\|^2 - \frac{1}{\gamma_i}\beta_i\|\phi_i\|^2 - \left(\frac{1}{2}\sigma_i\|x_i\|^2 - \max\lambda(P_i)\,\|x_i\|\,|\eta_i| + \frac{1}{2\sigma_i}\max\lambda^2(P_i)|\eta_i|^2\right)$$

$$+\frac{1}{2\sigma_i}\max\lambda^2(P_i)\,|\eta_i|^2$$

$$= -\frac{1}{2}\sigma_i\|x_i\|^2 - \frac{1}{\gamma_i}\beta_i\|\phi_i\|^2 - \left(\sqrt{\frac{\sigma_i}{2}}\|x_i\| - \frac{1}{\sqrt{2\sigma_i}}\sqrt{\max\lambda(P_i)}|\eta_i|\right)^2 + \frac{1}{2\sigma_i}\max\lambda^2(P_i)\,|\eta_i|^2$$

$$\le -\frac{1}{2}\sigma_i\|x_i\|^2 - \frac{1}{2\gamma_i}2\beta_i\|\phi_i\|^2 + \frac{1}{2\sigma_i}\max\lambda^2(P_i)\,|\eta_i|^2$$

Furthermore introducing a constant $\upsilon_i > 0$, the inequality above can be rewritten as

$$\dot{v}_i(x_i,\phi_i) \le -\frac{\upsilon_i}{2}\max\lambda(P_i)\|x_i\|^2 - \frac{\upsilon_i}{2\gamma_i}\|\phi_i\|^2 - (\sigma_i - \upsilon_i\max\lambda(P_i))\frac{\|x_i\|^2}{2}$$

$$-(2\beta_i - \upsilon_i)\frac{\|\phi_i\|^2}{2\gamma_i} + \frac{1}{2\sigma_i}\max\lambda^2(P_i)\,|\eta_i|^2$$

Let

$$\upsilon_i = \min\left[\frac{\sigma_i}{\max\lambda(P_i)},\ 2\beta_i\right] \tag{4-2-22}$$

and in view of Eqn.(4-2-9),

$$\dot{v}_i(x_i,\phi_i) \le -\upsilon_i v_i(x_i,\phi_i) + \frac{1}{2\sigma_i}\max\lambda^2(P_i)\rho_i^2 \tag{4-2-23}$$

For this first order differential inequality, the solution is

$$v_i(x_i,\phi_i) \le e^{-\upsilon_i t}\, v_i(x_i(0),\phi_i(0)) + \frac{1}{2\sigma_i}\max\lambda^2(P_i)\rho_i^2 \int_0^t e^{-\upsilon_i(t-u)}\,du$$

$$= e^{-\upsilon_i t} v_i(x_i(0),\phi_i(0)) + \frac{1}{2\upsilon_i\sigma_i}\max\lambda^2(P_i)\rho_i^2(1-e^{-\upsilon_i t}).$$

Hence conclusions (i) and (ii) of the theorem follow.

Similarly, consider

$$v(x,\Phi)=\sum_{i=1}^{n} v_i(x_i,\phi_i)$$

as a Lyapunov function for the overall system, in view of Eqn.(4-2-21),

$$\dot{v}(x,\Phi)=\sum_{i=1}^{n}\dot{v}_i(x_i,\phi_i)$$

$$\leq\sum_{i=1}^{n}\{-\upsilon_i v_i(x_i,\phi_i)+\frac{1}{2\sigma_i}\max\lambda^2(P_i)\rho_i^2\}$$

$$\leq -\min\upsilon_i v(x_i,\phi_i)+\sum_{i=1}^{n}\frac{1}{2\sigma_i}\max\lambda^2(P_i)\rho_i^2,$$

which results in (iii).

Corollary 4-1:

Associated with Theorem 4-1, the position tracking errors e_i are uniformly bounded by

$$|e_i| < (1+\alpha_i)\,\zeta_i, \tag{4-2-24}$$

where $\zeta_i>0$, is given by

$$\zeta_i=\sqrt{\frac{\max\lambda^2(P_i)\rho_i^2}{\upsilon_i\sigma_i\,\min\lambda(P_i)}-\frac{\phi_i^T\phi_i}{\gamma_i\min\lambda(P_i)}}.$$

Proof: From Eqn.(4-2-18) and Eqn.(4-2-20), it is known that x_i and ϕ_i are bounded by

$$x_i^T P_i x_i+\frac{1}{\gamma_i}\phi_i^T\phi_i<\frac{1}{\upsilon_i\sigma_i}\max\lambda^2(P_i)\rho_i^2.$$

Considering x_i only, it follows that:

$$\min\lambda(P_i)\|x_i\|^2 \leq x_i^T P_i x_i < \frac{1}{\upsilon_i \sigma_i} \max\lambda^2(P_i)\rho_i^2 - \frac{1}{\gamma_i}\phi_i^T \phi_i,$$

which means x_i is bounded by

$$\|x_i\| < \sqrt{\frac{\max\lambda^2(P_i)\rho_i^2}{\upsilon_i\sigma_i \min\lambda(P_i)} - \frac{\phi_i^T\phi_i}{\gamma_i \min\lambda(P_i)}} = \zeta_i$$

as ϕ_i is bounded. In view of Eqn.(4-2-6a) and Eqn.(4-2-12), $e_i=\alpha_i\varepsilon_i+\dot{\varepsilon}_i=\alpha_i x_{i1}+x_{i2}$, where x_{ij} with j=1,2 is the j-th component of x_i. As $|x_{ij}| \leq \|x_i\|$, for j=1 and 2, then

$$|e_i| \leq \alpha_i |x_{i1}| + |x_{i2}| < (1+\alpha_i)\,\zeta_i,$$

and the corollary is proved.

For Theorem 4-1, a geometrical interpretation is shown in Fig.4-2. The Lyapunov function $v_i(x_i, \phi_i)$ can be shown as a super ellipsoid defined above the super plane spanned by x_i, ϕ_i which has the dimensions of $2+z_i$. For a bounded initial condition $(x(0)_i, \phi(0)_i)$, $v_i(x_i, \phi_i)$ will move towards an interval $[0, \frac{1}{\upsilon_i\sigma_i}\max\lambda^2(P_i)\rho_i^2)$ because beyond this region $\dot{v}_i(x_i,\phi_i)$ is always negative definite. However, the system state trajectory $(x(t)_i, \phi(t)_i)$, which are the filtered tracking error and parameter estimation errors, can be shown by the projection of $v_i(x_i, \phi_i)$ on the (x_i, ϕ_i) super plane. Driven by the control forces, it tends to the residual set D_i given by the shaded area in the super plane. Geometrically, the residual set D_i is formed by a projection of the ellipsoid surface below the contour line $\frac{1}{\upsilon_i\sigma_i}\max\lambda^2(P_i)\rho_i^2$.

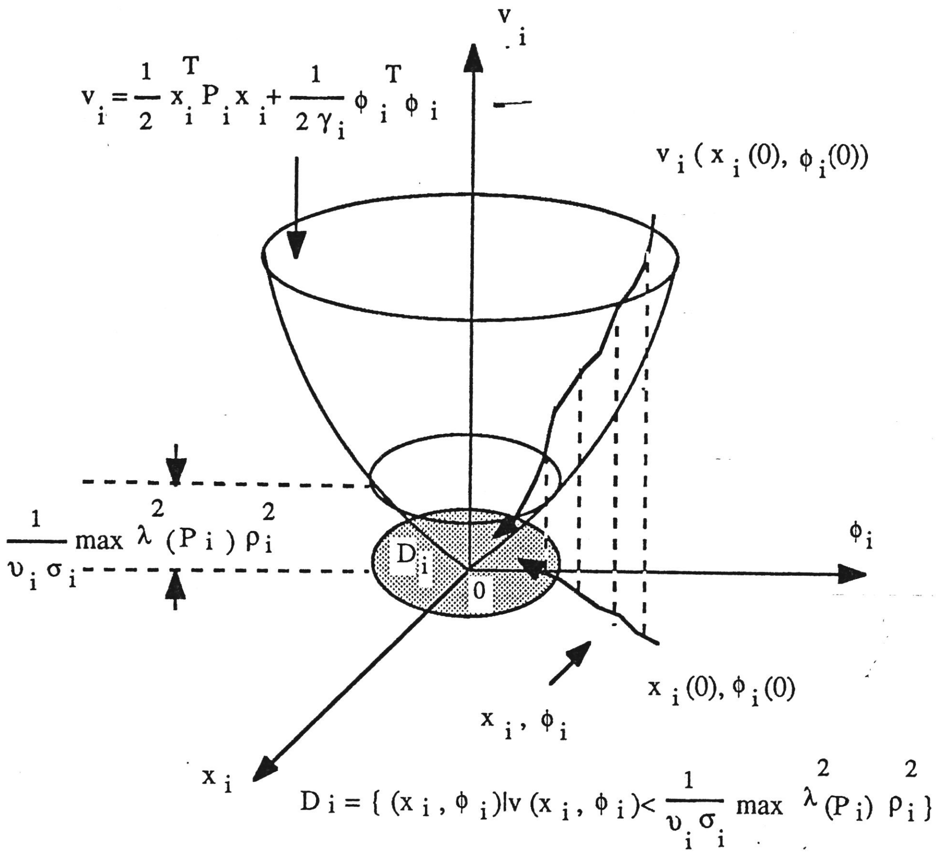

Fig.4-2. The geometrical interpretation of the system stability.

4-3. DISCUSSION

From Eqn.(4-2-17) and Fig.4-1, it can be seen that unlike a common decentralized system in which only local feedbacks are applied [27] [18], in the case here each subsystem is a multi-input single-output system which requires global feedbacks. It requires that all position and velocity information be shared by each joint and certainly makes the system more complicated. If the controlled robots have non-direct drive transmissions "pure" local feedback structures of large scale systems can be used since the high gear ratio in each transmission will reduce the coupling between subsystems significantly. However, when controlling direct drive robots, due to the strong interconnections between different subsystems local feedback may give poorer performance. Another consideration is that as robot systems are more compact than the usual large scale systems considered, ~~the~~ information exchange can be easily accomplished using the computer data bus.

Theorem 1 is a local result, i.e., it requires that the initial parameter estimates are inside a bounded range specified by Condition (C-5) (see 3-3-1). Condition (C-6) also gives the restrictions on the initial state errors $e(0)=\dot{e}(0)=0$. Unlike some dynamics systems in which the states are not measureable, the system states of robot dynamics (positions and velocities) are always measureable using tachogenerators and encoders so that this condition can always be satisfied by setting the starting points of the reference trajectories equal to the initial state of the robot system.

In the proof of Theorem 1, it is assumed that the ρ_i are bounded even though they are functions of e and $\dot{e}$. This assumption is based on the Conditions (C-1) – (C-6) since under these conditions the pure computed torque control law is able to give a bounded result. Theorem 1, however, gives some further insight into this boundedness. Even though the theorem does not prove that the boundedness of tracking errors in adaptive control is smaller than those of the pure computed torque scheme using the same a prori parameter estimates, the simulation results, which will be shown in Chapter 6, show that the adaptive control algorithm proposed gives significant improvements in tracking errors and robustness to payload change compared with the pure computed torque scheme. This means the residual set given by Theorem 1 is much smaller than that given by the pure computed torque scheme under the conditions (C-1) – (C-6).

In practice, for smaller tracking and estimation errors, the size of the residual set D_i should be as small as possible. From Eqn.(4-2-18), it can be seen that the size D_i is proportional to the bounds of the interaction and the disturbance terms ρ_i. For a certain ρ_i, in view of Eqn.(4-2-18), a smaller $\max\lambda(P_i)$ and a larger σ_i are expected to achieve a smaller D_i. However their determination is restricted by the Lyapunov equation Eqn.(4-2-

15). According to Eqn.(4-2-22), there are two options in choosing the controller parameter β_i:

(i)
$$2\beta_i > \frac{\sigma_i}{\max\lambda(P_i)}$$

which leads to

$$\upsilon_i = \frac{\sigma_i}{\max\lambda(P_i)} < 2\beta_i\,.$$

Then in view of Eqn.(4-2-18) it follows that the residual set becomes

$$D_i=\{(x_i,\ \phi_i)|v_i(x_i,\ \phi_i)<\frac{1}{2\sigma_i^2}\max\lambda^3(P_i)\rho_i^2\}; \tag{4-3-1}$$

(ii) In the case that

$$2\beta_i < \frac{\sigma_i}{\max\lambda(P_i)}$$

Eqn.(4-2-22) becomes

$$\upsilon_i = 2\beta_i < \frac{\sigma_i}{\max\lambda(P_i)}\,.$$

The residual set will be

$$D_i=\{(x_i,\ \phi_i)|v_i(x_i,\ \phi_i)<\frac{1}{4\beta_i\sigma_i}\max\lambda^2(P_i)\rho_i^2\}. \tag{4-3-2}$$

As

$$\frac{1}{2\beta_i} > \frac{\max\lambda(P_i)}{\sigma_i},$$

it follows that

$$\frac{1}{4\beta_i\sigma_i}\max\lambda^2(P_i)\rho_i^2 > \frac{1}{2\sigma_i^2}\max\lambda^3(P_i)\rho_i^2$$

This means the residual set Eqn.(4-3-2) is larger than Eqn.(4-3-1).

Since $\max\lambda(P_i)$ and $\sigma_i=\frac{1}{2}\min\lambda(L_i)$ are related by the Lyapunov equation $A_i^TP_i+P_iA_i=-Q_i$, quantitative analysis of the relationships between $\max\lambda(P_i)$, $\min\lambda(L_i)$ and A_i is quite difficult. Since Q_i and P_i influence the convergence of the Lyapunov function, β_i actually is a forgetting factor of the estimator, A_i ($-k_{vi}$ and $-k_{pi}$) gives ideal dynamic performance, the choice of parameters Q_i, P_i, β_i and A_i must be traded off in practical applications.

In most applications in which a robot hand is moved from one position to another following a certain trajectory, the initial values of the system state (positions, velocities and accelerations) are known. It is possible to set the initial values of the trajectory to equal the real values of the robot so that the initial tracking errors $e_i(0)$ and $\dot{e}_i(0)$ are both zeros. In these situations the tracking error x_i will stay in the set D_i.

Also, Theorem 4-1 only claims the paremeter estimation errors ϕ_i are bounded. Because of the existence of interactions and uncertainties these parameter estimates normally are not able to converge to their true values, i.e., the unbiased estimates cannot be obtained.

4-4. SUMMARY

In this Chapter, for robot tracking control, a novel adaptive approach based on the decentralized error system structure given in Chapter 3 was proposed. By means of introducing a linear filter the approach avoids the measurement of accelerations. For the decentralized subsystem the input of each subsystem is separated into two parts: the parameterized dominant input and non-dominant input. The adaptive controller is designed to compensate the dominant parts and the latter are treated as interconnections between the subsystems. In doing so only the diagonal elements of the inverse of the inertia matrix are required to be computed which makes the algorithm simple. Another advantage is that because of the two-component-controller architecture, in which the non-adaptive control law is fixed by a set of parameter estimates, the algorithm ensures the existence of the inverse of the estimated inertia matrix. The tracking errors are adjusted by the adaptive control component. The tool used in design is the Lyapunov direct method and so a further advantage of this method is that quantitative results about bounded position tracking errors are available. Theorem 4-1 is proved to demonstrate this, and a geometrical illustration of stability is given.

However, because the error states are the filtered position and velocity errors, quantitative velocity error boundedness could not be obtained directly. Also Theorem 4-1 needs the Conditions (C-3) – (C-6) given in Section 3-3-1 in addition to the assumptions made in Chapters 2 and 3 to ensure the interconnections are bounded. In order to take advantage of this, a further algorithm is investigated in Chapter 5. In this investigation, the bounds of the interconnections are treated as functions of the system states which represent more realistic descriptions of robot systems. In this algorithm, Conditions (C-3) to (C-6) are removed.

Chapter 5.

ROBUST ADAPTIVE CONTROLLER METHOD – 2

5-1. INTRODUCTION

In the robust adaptive controller shown in Chapter 4, the avoidance of acceleration measurements is achieved by introducing a linear operator $L^{-1}(s)$ to both sides of Eqn.(3-3-14b). However, as the system state of Eqn.(4-2-13) is defined by the filtered error states ε_i and $\dot{\varepsilon}_i$ (see Eqn.(4-2-12)) instead of the original error states e_i and $\dot{e}_i$ (position and velocity errors), the boundedness of the position errors cannot be shown directly by Theorem 4-1. (Although it has been proved by Corollary 4-1). Most importantly, as the interconnections η_i depend on the q and $\dot{q}$ of the overall system, in order to prove the boundednesses of filtered position errors, Conditions (C-3) – (C-6) in Chapter 3 are required to ensure those interconnections are all bounded. These are quite restrictive conditions.

In this chapter improvements on this approach will be made so that the algorithm presented in Section 4-2 can be simplified and a direct result on the quantitative boundednesses of both the position errors e_i and the velocity errors $\dot{e}_i$ can be obtained. More significantly, the restrictions of Conditions (C-3) – (C-6) will be removed in this algorithm. The ideas are based on the Kalman-Yacubovitch lemma (positive real lemma) [33] [64] and the adaptive control of decentralized system with interconnections proposed by [27]. According to the properties of the strictly positive real theory, it is known that a necessary condition for a transfer function to be strictly positive real is that the number of its zeros should be either equal to, or less than the number of its poles by one. Since the error dynamic equation Eqn.(3-3-14b) has two poles and no zero, an additional zero is required to satisfy this condition. In Chapter 4 this is achieved by introducing the output matrix b^TP which is a natural result of solving the Lyapunov equation. In the method proposed by this chapter, the Hurwitz linear operator proposed by Narendra and Valavani [48] is used to operate on the right hand side of Eqn.(3-3-14) to get this zero for the error equations. In this decentralized system formulation the position and velocity errors appear explicitly in each subsystem. It can be shown that the interconnections among the subsystems are bounded by the overall system states and this leads to a standard decentralized system structure given by [27] and [53]. Using the same idea of adaptive controller design for the decentralized system proposed by [27] a robust adaptive control algorithm can be derived which will give bounded position and velocity tracking errors. The results also show the relationship between these boundednesses and system uncertainties and reference trajectories.

This chapter is organized as follows. In Section 5-2, the linear operator $P_L(\theta)$ and its properties given by [48] is introduced. Section 5-3 will show that by using this operator in the right hand of Eqn.(3-3-14b) a state realization can be obtained which satisfies the strictly positive real condition. In Section 5-4, the robust control algorithm is stated and the main result will be shown by Theorem 5-1. Since the magnitudes of the control signals are of concern, in Section 5-5 a proof on the bounded control magnitudes is presented. Section 5-6 consists of discussions on the method and finally in Section 5-7 a summary is given.

5-2. LINEAR OPERATOR $P_L(\theta)$ AND ITS PROPERTIES

Using the linear operator L(s) (Hurwitz polynomial, see Appendix A)given by Section 4-2-1, a new linear operator

$$P_L(\theta)=L(s)\theta(t)L^{-1}(s) \qquad (5\text{-}2\text{-}1)$$

proposed by [48] is introduced. In this operator $\theta(t)$ is a bounded differentiable function of time. As shown in [48], $P_L(\theta)$ has the following properties:

1). $P_L(\theta(t))$ is a linear function in $\theta(t)$, i.e.,

$$P_L(c\theta) = L(s)c\theta L^{-1}(s) = cL(s)\theta(t)L^{-1}(s) = cP_L(\theta)$$

and

$$P_L(c_1\theta_1+c_2\theta_2)=L(s)c_1\theta_1L^{-1}(s)+L(s)c_2\theta_2L^{-1}(s)$$

$$=c_1L(s)\theta_1L^{-1}(s)+c_2L(s)\theta_2L^{-1}(s)$$

$$=c_1P_L(\theta_1)+c_2P_L(\theta_2)$$

for real constants c, c_1 and c_2.

2). If $\theta=\bar{\theta}$, which is a constant, then

$$P_L(\bar{\theta})=\bar{\theta}, \tag{5-2-2}$$

as, $P_L(\bar{\theta})=L(s)\bar{\theta}L^{-1}(s)=\bar{\theta}L(s)L^{-1}(s)=\bar{\theta}$.

3). If $L(s)=s+\alpha$, then $P_L(\theta(t))=[\theta(t)+\dot{\theta}(t)L^{-1}(s)]$.

Property 3) above can be proved as follows. Let f(t) be a differentiable function in t, and

$$g(t)=L^{-1}(s)f(t)=\frac{1}{s+\alpha}f(t). \tag{5-2-3}$$

Then it follows that

$$P_L(\theta(t))f(t)=L(s)\theta(t)L^{-1}(s)f(t)$$

$$=L(s)\theta(t)g(t)$$

$$=(s+\alpha)\theta(t)g(t)$$

$$=\dot{\theta}(t)g(t)+\theta(t)\dot{g}(t)+\alpha\theta(t)g(t)$$

$$=\dot{\theta}(t)L^{-1}(s)f(t)+\theta(t)[\dot{g}(t)+\alpha g(t)]$$

In view of Eqn.(5-2-3), it also follows that $\dot{g}(t)+\alpha g(t)=f(t)$, so

$$P_L(\theta(t))f(t)=\dot{\theta}(t)L^{-1}(s)f(t)+\theta(t)f(t)$$

$$=[\dot{\theta}(t)L^{-1}(s)+\theta(t)]f(t),$$

which implies that $P_L(\theta(t))=[\theta(t)+\dot{\theta}(t)L^{-1}(s)]$.

3). If $w_o(s)$ is a stable transfer function with two poles and null zero, i.e., $w_o(s)=\frac{1}{s^2+a_1s+a_2}$, $L(s)=s+b$ is a Hurwitz polynomial of degree 1, then $w_o(s)L(s)$ is a stable transfer function.

In this study, only the first order operator $L(s)=s+\alpha$ and its inverse $L^{-1}(s)=\frac{1}{s+\alpha}$ is of interest.

5-3. SYSTEM STRUCTURE

The dynamic equation Eqn.(3-2-3)

$$D(q)\ddot{q}+h(q,\dot{q})+g(q)+d_o=u \qquad (5\text{-}3\text{-}1)$$

will be considered in this chapter again. For d_o, in this chapter, a new assumption is made to replace Assumption 3-5) made in Chapter 3:

Assumption 5-1: Term $d_o \in R^n$ represents the uncertainties in parameters and structure of the robot dynamic system. It represents the interconnections among different subsystems in such a way that each component of it is a function of the velocities of the overall system, i.e.,

$$d_o=d_o(\dot{q})=[d_{o1}(\dot{q})\ d_{o2}(\dot{q})\ \dots\ d_{on}(\dot{q})]^T$$

Furthermore, it is assumed that the boundedness of its i-th component $d_{oi}=d_{oi}(\dot{q})$ depends on the boundedness of $\dot{q}$, i.e., there exist some positive constants a'_{ij} such that

$$|d_{oi}| \le \sum_{j=1}^{n} a'_{ij} |\dot{q}_j|, \qquad \text{for } i=1,2,...,n. \qquad (5\text{-}3\text{-}2)$$

Physically, it is used to model uncertain torques such as the ignored friction torque which may be linear or nonlinear functions of $\dot{q}$, etc..

Based on equation of motion Eqn.(5-3-1), the decentralized system structure will be explored in Section 5-3-1. In Section 5-3-2, the properties of the interconnection among the subsystems will be discussed.

5-3-1. Decentralized Error System

For the equation of motion (5-3-1), the input torque u is given by:

$$u=u_1-u_2, \qquad (5\text{-}3\text{-}3)$$

where

$$u_1=\hat{D}(q)[\ddot{q}_d+K_v(\dot{q}_d-\dot{q})+K_p(q_d-q)]+\hat{h}(q,\dot{q})+\hat{g}(q) \qquad (5\text{-}3\text{-}4)$$

is a non-adaptive component, and

$$u_2=\hat{D}(q)u_a \qquad (5\text{-}3\text{-}5)$$

is an adaptive control component. In the Eqn.(5-3-4), $K_v=\text{diag}\{k_{vi}\}\in R^{nxn}$, $K_p=\text{diag}\{k_{pi}\}\in R^{nxn}$ with k_{vi}, $k_{pi}>0$, for i=1,2,...,n. According to Eqns. (5-3-4) and (5-3-5), Eqn.(5-3-3) can be written as:

$$u=\hat{D}(q)[\ddot{q}_d+K_v(\dot{q}_d-\dot{q})+K_p(q_d-q)-u_a]+\hat{h}(q,\dot{q})+\hat{g}(q). \qquad (5\text{-}3\text{-}6)$$

Substitution of Eqn.(5-3-6) into Eqn.(5-3-1), leads to

$$\ddot{e}+K_v\dot{e}+K_pe=\hat{D}^{-1}(q)[-\ \tilde{D}(q)\ddot{q}\ -\tilde{h}(q,\dot{q})-\tilde{g}(q)-d_o]+u_a. \qquad (5\text{-}3\text{-}7)$$

In Eqn.(5-3-7), $e=q_d-q$ is the position error; $\hat{D}^{-1}(q)$ is the inverse of estimated inertia matrix $\hat{D}(q)$; $\tilde{D}(q)=\hat{D}(q)-D(q)$, $\tilde{h}(q,\dot{q})=\hat{h}(q,\dot{q})-h(q,\dot{q})$ and $\tilde{g}(q)=\hat{g}(q)-g(q)$.

The i-th error subsystem of Eqn.(5-3-7) is given by

$$\ddot{e}_i+k_{vi}\dot{e}_i+k_{pi}e_i= - \bar{\theta}_i^T\hat{J}_{ii}(q)\omega_i(q,\dot{q},\ddot{q})+u_{ai}-d_i. \qquad (5\text{-}3\text{-}8)$$

where $\bar{\theta}_i$, $\hat{J}_{ii}(q)$ and $\omega_i(q,\dot{q},\ddot{q})$ are the same as those in Eqn.(3-3-14b). It is worth noting that the right hand side of Eqn.(5-3-8) differs from Eqn.(3-3-14b) in that each term has an opposite sign caused by different definitions of Eqn.(5-3-3) and the parameter estimation errors. For instance, in this chapter estimation error of inertia matrix is defined by $\tilde{D}(q)=\hat{D}(q)-D(q)$ rather than $\tilde{D}(q)=D(q)-\hat{D}(q)$ as used in Chapter 3. The reason for this is that it will make the stability analysis (the proof of Theorom 5-1 which will be given in Section 5-4-2) easier.

Recalling Eqn.(3-3-12), the last term in the right hand side of Eqn.(5-3-8), which represents the interconnections among the different subsystems, can be written as

$$\begin{aligned} d_i &= d_i(q,\dot{q},\ddot{q}) \\ &= \sum_{j=1 j\neq i}^{n} \sum_{k=1}^{n} \hat{J}_{ij}(q) \tilde{D}_{jk}(q)\ddot{q}_k + \sum_{j=1 j\neq i}^{n} \hat{J}_{ij}(q)\tilde{h}_j(q,\dot{q}) + \sum_{j=1 j\neq i}^{n} \hat{J}_{ij}(q)\tilde{g}_j(q) + \sum_{j=1}^{n} \hat{J}_{ij}(q)d_{oj} \end{aligned} \qquad (5\text{-}3\text{-}9)$$

where $\hat{J}_{ij}(q)$ is the i-j-th element of $\hat{J}(q)=\hat{D}^{-1}(q)$ and d_{oj} is the j-th component of d_o. As a disturbance presented in Eqn.(5-3-9), term $d_i(q,\dot{q},\ddot{q})$ represents the influence on the error states of subsystem i caused by the states of the other subsystems. It indicates that the tracking errors of joint i will depend on the positions, velocities and accelerations of all other joints of the robot arms.

For subsystem i, the linear operator $P_{Li}(\bar{\theta}_i)$ defined by Eqn.(5-2-1) is applied. From property 2) of $P_{Li}(\bar{\theta}_i)$ (see Eqn.(5-2-2)), it follows that $\bar{\theta}_i=P_{Li}(\bar{\theta}_i)=L_i(s)\bar{\theta}_i L_i^{-1}(s)$ since $\bar{\theta}_i$ is a constant vector. As a result of this, Eqn.(5-3-8) can be written as

$$\ddot{e}_i+k_{vi}\dot{e}_i+k_{pi}e_i=-L_i(s)\bar{\theta}_i^T L_i^{-1}(s)\hat{J}_{ii}(q)\omega_i(q,\dot{q},\ddot{q})+u_{ai}-d_i. \tag{5-3-10}$$

Specifying that

$$L_i(s)=s+\alpha_i, \tag{5-3-11}$$

where $\alpha_i>0$, is a constant, then Eqn.(5-3-10) can be written as

$$\ddot{e}_i+k_{vi}\dot{e}_i+k_{pi}e_i=-L_i(s)\bar{\theta}_i^T\delta_i(q,\dot{q})+u_{ai}-d_i, \tag{5-3-12}$$

where

$$\begin{aligned}\delta_i(q,\dot{q})&=L_i^{-1}(s)\hat{J}_{ii}(q)\omega_i(q,\dot{q},\ddot{q})\\&=\frac{1}{s+\alpha_i}\hat{J}_{ii}(q)\omega_i(q,\dot{q},\ddot{q}).\end{aligned} \tag{5-3-13}$$

As shown in Section 4-2-1, $\delta_i(q,\dot{q})$ above does not depend on $\ddot{q}$ explicitly and there is no need to measure the accelerations in the implementation of $\delta_i(q,\dot{q})$.

This suggests specifying u_{ai} in the form of

$$u_{ai}=L_i(s)\hat{\theta}_i^T\delta_i(q,\dot{q}), \tag{5-3-14}$$

where $\hat{\theta}_i$ is an estimate vector for θ_i, and the design objective is to find out an updating law for $\hat{\theta}_i$ so that the error states in Eqn.(5-3-12) will become as small as possible.

Substituting Eqn.(5-3-14) into Eqn.(5-3-12) results in

$$\ddot{e}_i+k_{vi}\dot{e}_i+k_{pi}e_i=L_i(s)[\phi_i^T\delta_i(q,\dot{q})-\eta_i], \tag{5-3-15}$$

where $\phi_i=\hat{\theta}_i-\theta_i$ (which is different from the definition of $\phi_i=\theta_i-\hat{\theta}_i$ made in Chapter 4) is the estimation error vector and

$$\eta_i=L_i^{-1}(s)d_i \tag{5-3-16}$$

in which $L_i^{-1}(s)=\frac{1}{s+\alpha_i}$.

5-3-2. Properties of Interconnection η_i

According to Eqn.(5-3-9), η_i can be written as

$$\eta_i=\eta_{i1}+\eta_{i2}, \tag{5-3-17}$$

where

$$\eta_{i1}=\frac{1}{s+\alpha_i}\sum_{j=1 j\neq i}^{n}\sum_{k=1}^{n}\hat{J}_{ij}(q)\ \bar{D}_{jk}(q)\ \ddot{q}_k$$

$$=\frac{1}{s+\alpha_i}g_{i1}(t), \tag{5-3-18}$$

$$\eta_{i2}=\frac{1}{s+\alpha_i}[\sum_{j=1 j\neq i}^{n}\hat{J}_{ij}(q)\bar{h}_j(q,\dot{q})+\sum_{j=1 j\neq i}^{n}\hat{J}_{ij}(q)\bar{g}_j(q)+\sum_{j=1}^{n}\hat{J}_{ij}(q)d_{oj}(\dot{q})],$$

$$=\frac{1}{s+\alpha_i}g_{i2}(t), \tag{5-3-19}$$

with

$$g_{i1}(t)=\sum_{j=1 j\neq i}^{n}\sum_{k=1}^{n}\hat{J}_{ij}(q)\ \bar{D}_{jk}(q)\ \ddot{q}_k, \tag{5-3-20}$$

$$g_{i2}(t)=\sum_{j=1 j\neq i}^{n}\hat{J}_{ij}(q)\bar{h}_j(q,\dot{q})+\sum_{j=1 j\neq i}^{n}\hat{J}_{ij}(q)\bar{g}_j(q)+\sum_{j=1}^{n}\hat{J}_{ij}(q)d_{oj}(\dot{q}). \tag{5-3-21}$$

For $\hat{J}_{ij}(q)$, $\bar{D}(q)_{ij}$, $\bar{g}_i(q)$ and $\bar{h}_i(q,\dot{q})$ in the equations above, the following lemma applies:

Lemma 5-1. $\hat{J}_{ij}(q)$, $\bar{D}(q)_{ij}$ and $\bar{g}_i(q)$ are bounded by some constants and the boundedness of $\bar{h}_i(q,\dot{q})$ depends on the boundedness of $|\dot{q}_k|$ (k=1,2,...,n), i.e., there exist some positive constants j'_{ij}, d'_{ij}, g'_i and h'_{ik} such that:

$$|\hat{J}_{ij}(q)| \le j'_{ij}, \tag{5-3-22}$$

$$|\ \bar{D}_{jk}(q)\ | \le d'_{ij}, \tag{5-3-23}$$

$$|\bar{g}_i(q)| \le g'_i, \tag{5-3-24}$$

$$|\bar{h}_i(q,\dot{q})| \le \sum_{k=1}^{n} h'_{ik}|\dot{q}_k|. \tag{5-3-25}$$

Proof: According to Assumption 3-3), $\hat{D}(q)$ is positive definite and so is $[\hat{D}(q)]^{-1}$. Moreover since all joints of the robot arms under consideration are revolute (see Assumption 2-1)), each element of $[\hat{D}(q)]^{-1}$ consists of trigonometric functions of q only, so $\hat{J}_{ij}(q)$, the i-j-th element of $[\hat{D}(q)]^{-1}$, must be bounded. This means that Eqn.(5-3-22) holds. Eqns.(5-3-23) – (5-3-24) arise because $D_{ij}(q)$, $g_i(q)$, $\hat{D}_{ij}(q)$ and $\hat{g}_i(q)$ are all bounded, i.e.,

$$\begin{aligned}|\bar{D}_{ij}(q)| &\le |D_{ij}(q)-\hat{D}_{ij}(q)|\\ &\le |D_{ij}(q)| + |\hat{D}_{ij}(q)|\\ &\le d^o_{ij}+d^*_{ij}\\ &=d'_{ij},\end{aligned}$$

according to Eqns.(2-2-16) and (3-2-9). Based on Eqns.(2-2-17) and (3-2-11)

$$\begin{aligned}|\bar{g}_i(q)| &\le |g_i(q)-\hat{g}_i(q)|\\ &\le |g_i(q)| + |\hat{g}_i(q)|\\ &\le g^o_i+g^*_i\\ &=g'_i.\end{aligned}$$

Furthermore, for Eqn.(5-3-25), it follows that, according to Eqns.(2-2-19) and (3-2-10),

$$
\begin{aligned}
|\bar{h}_i(q,\dot{q})| &\le |h_i(q,\dot{q}) - \hat{h}_i(q,\dot{q})| \\
&\le \sum_{k=1}^{n} |h^o_i - h^*_i| \, |\dot{q}_k| \\
&\le \sum_{k=1}^{n} (h^o_i + h^*_i) \, |\dot{q}_k|. \\
&= \sum_{k=1}^{n} h'_{ik} |\dot{q}_k|,
\end{aligned}
$$

and Lemma 5-1 follows.

Lemma 5-2. The interconnection η_i, given by Eqn.(5-3-17), is bounded by the overall system states q_k, $\dot{q}_k$ (for k=1, 2,..., n) in such a way that there exist some positive constants a_{ik}, b_{ik} and c_{ik} so that

$$
|\eta_i| \le \sum_{k=1}^{n} a_{ik}|q_k| + b_{ik}|\dot{q}_k| + c_{ik}. \tag{5-3-26}
$$

Proof: Firstly, the boundedness of $g_{i1}(t)$ and $g_{i2}(t)$ are shown to depend on the system states. According to Eqn.(5-3-20) and Lemma 5-1, the boundedness of $g_{i1}(t)$ satisfies

$$
\begin{aligned}
|g_{i1}(t)| &\le \sum_{j=1 j\ne i}^{n} \sum_{k=1}^{n} |\hat{J}_{ij}(q)| \, |\bar{D}(q)_{jk}(q)| \, |\ddot{q}_k| \\
&\le \sum_{j=1 j\ne i}^{n} \sum_{k=1}^{n} j'_{ij} \, d'_{jk} \, |\ddot{q}_k| \\
&= \sum_{k=1}^{n} \sum_{j=1 j\ne i}^{n} j'_{ij} \, d'_{jk} \, |\ddot{q}_k| \\
&= \sum_{k=1}^{n} b''_{ik} \, |\ddot{q}_k|,
\end{aligned} \tag{5-3-27}
$$

where

$$
b''_{ik} = \sum_{j=1 j\ne i}^{n} j'_{ij} \, d'_{jk}.
$$

For Eqn.(5-3-21), using Assumption 5-1) (see Eqn.(5-3-2)) and Lemma 5-1, it is given that:

$$|g_{i2}(t)| \le \Big| \sum_{j=1 j\neq i}^{n} \hat{J}_{ij}(q)\tilde{h}_j(q,\dot{q})\Big| + \Big| \sum_{j=1 j\neq i}^{n} \hat{J}_{ij}(q)\bar{g}_j(q)\Big| + \Big| \sum_{j=1}^{n} \hat{J}_{ij}(q) d_{oj}(\dot{q}) \Big|$$

$$\le \sum_{j=1 j\neq i}^{n} |\hat{J}_{ij}(q)|\, |\tilde{h}_j(q,\dot{q})| + \sum_{j=1 j\neq i}^{n} |\hat{J}_{ij}(q)|\, |\bar{g}_j(q)| + \sum_{j=1}^{n} |\hat{J}_{ij}(q)|\, |d_{oj}(\dot{q})|$$

$$\le \sum_{j=1 j\neq i}^{n} j'_{ij} \Big(\sum_{k=1}^{n} h'_{jk}|\dot{q}_k|\Big) + \sum_{j=1 j\neq i}^{n} j'_{ij}\, g'_j + \sum_{j=1}^{n} j'_{ij} \sum_{k=1}^{n} a'_{ik}|\dot{q}_k|$$

$$= \sum_{k=1}^{n} \sum_{j=1}^{n} j'_{ij}\, (h'_{jk}+a'_{ik})|\dot{q}_k| - \sum_{k=1}^{n} j'_{ii} h'_{ik}|\dot{q}_k| + \sum_{j=1 j\neq i}^{n} j'_{ij}\, g'_j$$

$$= \sum_{k=1}^{n} \Big[\sum_{j=1}^{n} j'_{ij}\, (h'_{jk}+a'_{ik}) - j'_{ii} h'_{ik}\Big]\, |\dot{q}_k| + \sum_{j=1 j\neq i}^{n} j'_{ij}\, g'_j$$

$$\le \sum_{k=1}^{n} a''_{ik}|\dot{q}_k| + c''_i \tag{5-3-28}$$

where

$$a''_{ik} = \sum_{j=1}^{n} j'_{ij}\, (h'_{jk}+a'_{ik}) + j'_{ii} h'_{ik}, \tag{5-3-29}$$

$$c_i'' = \sum_{j=1 j\neq i}^{n} j'_{ij}\, g'_j \tag{5-3-30}$$

are positive constants.

Secondly, it can be shown that the boundedness of η_i also depends on q and $\dot{q}$. As the operator $L_i^{-1}(s)=\frac{1}{s+\alpha_i}$ is a linear exponentially stable transfer function and its impulse-response is $h_i(t) = e^{-\alpha_i t}$, then it follows from Eqns.(5-3-18) and (5-3-19) that

$$\eta_{im} = h_i(t) * g_{im}(t)$$

$$= \int_0^t h_i(t-\tau)\, g_{im}(\tau) d\tau, \qquad \text{for } m=1,\ 2.$$

where (*) is the convolutional integral operation, and

$$|\eta_{im}| \le \int_0^t |h_i(t-\tau)|\,|g_{im}(\tau)|d\tau, \qquad \text{for } m=1,\,2. \tag{5-3-31}$$

Using Eqns.(5-3-31) and (5-3-27), the boundedness of η_{i1} satisfies the inequality:

$$|\eta_{i1}| \le \int_0^t |e^{-\alpha_i(t-\tau)}| \sum_{j=1 j\neq i}^{n} \sum_{k=1}^{n} |\hat{J}_{ij}(q(\tau))|\,|\quad \bar{D}_{jk}(q(\tau))|\,|\ddot{q}_k(\tau)|\, d\tau$$

$$\le \sum_{k=1}^{n} b''_{ik} \int_0^t |e^{-\alpha_i(t-\tau)}|\,|\ddot{q}_{ik}(\tau)|\, d\tau$$

$$\le \sum_{k=1}^{n} b''_{ik} \int_0^t |d\dot{q}_k(\tau)|\,.$$

As the integrated function

$$|d\dot{q}_k| = \begin{cases} d\dot{q}_k & \text{if } d\dot{q}_k \ge 0 \\ -d\dot{q}_k & \text{if } d\dot{q}_k < 0 \end{cases},$$

the integration given above exists and is bounded by $|\dot{q}_k|$, i.e.,

$$\int_0^t |d\dot{q}_k(\tau)| \le \overset{o}{b}_k\,(|\dot{q}_k| + |\dot{q}_k(0)|).$$

where $\overset{o}{b}_k$ are positive constants. Then it follows that

$$|\eta_{i1}| \le \sum_{k=1}^{n} b''_{ik} \int_0^t |d\dot{q}_k(\tau)|\,.$$

$$\le \sum_{k=1}^{n} b_{ik} \quad |\dot{q}_k| + c'_{oi}. \tag{5-3-32}$$

where $b_{ik}=b''_{ik}\overset{o}{b}_k$, and $c'_{oi} = \sum_{k=1}^{n} b''_{ik}\overset{o}{b}_k\,|\dot{q}_k(0)|$ is a constant related to the initial condition of $|\dot{q}_k(0)|$.

Similarly, for η_{i2}, in view of Eqns.(5-3-8),

$$|\eta_{i2}| \le \int_0^t |e^{-\alpha_i(t-\tau)}|\,|g_{i2}(\tau)|d\tau.$$

According to Eqn.(5-3-28), it follows that

$$|\eta_{i2}| \leq \sum_{k=1}^{n} \int_{o}^{t} |e^{-\alpha_i(t-\tau)}|\, a''_{ik}|\,\dot{q}_k|d\tau + \int_{o}^{t} e^{-\alpha_i(t-\tau)}\, c_i''d\tau$$

$$\leq \sum_{k=1}^{n} a''_{ik} \int_{o}^{t} |e^{-\alpha_i(t-\tau)}|\, |dq_k| + \frac{c''_i}{\alpha_i}(e^{-\alpha_i t} -1)$$

$$\leq \sum_{k=1}^{n} a''_{ik} \int_{o}^{t} |dq_k| + \frac{c''_i}{\alpha_i}$$

$$\leq \sum_{k=1}^{n} a_{ik}\, |q_k| + c''_{oi}, \tag{5-3-33}$$

where a_{ik} and c''_{oi} are positive constants. In view of Eqns.(5-3-17), (5-3-32) and (5-3-33), and letting $c_i = c'_{oi}+c''_{oi}$, it follows that

$$|\eta_i| \leq |\eta_{i1}|+|\eta_{i2}|$$

$$\leq \sum_{k=1}^{n}(a_{ik}\,|q_k| +b_{ik}\,|q_k|)+c'_{oi}+c''_{oi}$$

$$\leq \sum_{k=1}^{n}(a_{ik}\,|q_k| +b_{ik}\,|q_k|)+c_i.$$

Further, denoting $c_i = \sum_{k=1}^{n} c_{ik}$, the inequality above becomes Eqn.(5-3-26) and the lemma follows.

5-4. ROBUST ADAPTIVE CONTROLLER

In this section, the algorithm of the robust adaptive controller will be presented. In Section 5-4-1, a state space realization of the error system is given. This structure is a set of decentralized subsystems in which the overall system states q and $\dot{q}$ interconnect each system. Further investigation will show that the interconnections depend on the boundedness of q and $\dot{q}$ which is an important condition in adaptive controller design and stability analysis. For the state space realization, a proper choice of some system design parameters will give a strictly positive real transfer function. Based on the results given in Section 5-4-1, a robust adaptive update law will be derived using the Lyapunov direct method. The main results are the quantitative convergence rate and boundedness of the position and velocity tracking errors.

In the analysis which follows, a necessary lemma is stated as follows:

Lemma 5-3. For a scalar η^o satisfying $|\eta^o| \le a\,|\psi_1|+b\,|\psi_2|$, where a and b are some positive constants, there exists a constant $c \ge \sqrt{a^2+b^2}$ such that the following inequality holds:

$$|\eta^o| \le a\,|\psi_1|+b\,|\psi_2| \le c\,\sqrt{(\psi_1{}^2+\psi_2{}^2)} = c\,\|\psi\|, \qquad (5\text{-}4\text{-}1)$$

where $\|\psi\|$ is the Euclidean norm of vector $\psi = [\psi_1 \;\; \psi_2]^T$ defined by the equality above.

Proof: In view of Eqn.(5-4-1), the central term can be written as

$$(a\,|\psi_1|+b\,|\psi_2|)^2 \le c^2\,(\psi_1{}^2+\psi_2{}^2)$$

which is equivalent to

$$c^2\,(\psi_1{}^2+\psi_2{}^2) - a^2\,\psi_1{}^2 - 2ab|\psi_1|\,|\psi_2| - b^2\,\psi_2{}^2$$

$$= (c^2 - a^2)\psi_1{}^2+(c^2 - b^2)\psi_2{}^2 - 2ab\,|\psi_1|\,|\psi_2|$$

$$\ge 0.$$

By completing the square, the left hand side equals:

$$[(c^2 - a^2)\psi_1{}^2 - 2\sqrt{(c^2-a^2)(c^2-b^2)}\,|\psi_1|\,|\psi_2|+(c^2 - b^2)\psi_2{}^2]$$

$$+2\sqrt{(c^2-a^2)(c^2-b^2)}\,|\psi_1|\,|\psi_2| - 2ab|\psi_1|\,|\psi_2|$$

$$=[\sqrt{c^2-a^2}\,|\psi_1| - \sqrt{c^2-b^2}\,|\psi_2|]^2+2\sqrt{(c^2-a^2)(c^2-b^2)}\,|\psi_1|\,|\psi_2| - 2ab|\psi_1|\,|\psi_2| \ge 0.$$

Let $c \ge a$, $c \ge b$, then $[\sqrt{c^2-a^2}\,|\psi_1| - \sqrt{c^2-b^2}\,|\psi_2|]^2$ will be real and always greater than zero. Moreover the inequality will hold if

$$2\sqrt{(c^2-a^2)(c^2-b^2)}\,|\psi_1|\,|\psi_2| - 2ab|\psi_1|\,|\psi_2| \ge 0.$$

This gives

$$(c^2-a^2)(c^2-b^2)-a^2b^2$$

$$=c^4-a^2c^2-b^2c^2$$

$$=c^2(c^2-a^2-b^2)$$

$$\geq 0$$

and

$$c^2\geq a^2+b^2.$$

As $a^2+b^2\geq \max(a^2, b^2)$, the conditions $c\geq a$ and $c\geq b$ are included in condition $c\geq\sqrt{a^2+b^2}$ and Lemma 5-3 is proved.

5-4-1. State Space Realization

Eqn.(5-3-15) can be written in terms of the operator s as:

$$(s^2+k_{vi}s+k_{pi})e_i=(s+\alpha_i)[\phi_i^T\delta_i(q,\dot{q})-\eta_i]. \tag{5-4-2}$$

The transfer function is

$$w_i(s)=w_{oi}(s)L_i(s)=\frac{s+\alpha_i}{s^2+k_{vi}s+k_{pi}} \tag{5-4-3}$$

where $w_{oi}(s)$ is the transfer function of the original system Eqn.(5-3-8) and $L_i(s)$, given byEqn.(5-3-11),introduces an auxiliary zero to the closed system.

To express Eqn.(5-4-2) in tracking error state space, a 2-dimension state vector can be defined as:

$$y_i = \begin{bmatrix} e_i \\ \dot{e}_i \end{bmatrix}. \tag{5-4-4}$$

Correspondingly, in robot joint space, the position and velocity vector is given by

$$y_{qi} = \begin{bmatrix} q_i \\ \dot{q}_i \end{bmatrix}, \tag{5-4-5}$$

and the reference trajectory vector by

$$y_{di} = \begin{bmatrix} q_{di} \\ \dot{q}_{di} \end{bmatrix}. \tag{5-4-6}$$

According to Eqn.(5-4-4), the error equation Eqn.(5-3-15) can be expressed by a realization:

$$\dot{y}_i = A_i y_i + b_i \phi_i^T \delta_i(q,\dot{q}) - b_i \eta_i \tag{5-4-7}$$

$$e_i = h_i^T y_i, \tag{5-4-8}$$

where

$$A_i = \begin{bmatrix} 0 & 1 \\ -k_{pi} & -k_{vi} \end{bmatrix}, \tag{5-4-9a}$$

$$b_i = \begin{bmatrix} 1 \\ \alpha_i - k_{vi} \end{bmatrix}, \tag{5-4-9b}$$

$$h_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{5-4-9c}$$

It has been shown, by Lemma 5-2 in Section 5-3, that the interconnection η_i in Eqn.(5-4-7) satisfies Eqn.(5-3-26). Furthermore it can be shown that for η_i the following corollary applies:

Corollary 5-1. The ~~boundedness~~boundnesses of η_i in Eqn.(5-4-7) is related to the boundednesses of positions and velocities of subsystem $y_{qi} = [\, q_i \,, \dot{q}_i \,]$, for $i=1, 2, ..., n$, in such a way that there exist constants $\chi_{ik} \geq \sqrt{a_{ik}^2 + b_{ik}^2}$ such that

$$|\eta_i| \leq \sum_{k=1}^{n} \chi_{ik}\|y_{qk}\|+c_{ik}. \tag{5-4-10}$$

Proof: Lemma 5-3 is used directly by replacing a, b, c, ψ_1, ψ_2 and ψ in Eqn.(5-4-1) by a_{ik}, b_{ik}, χ_{ik}, q_k, $\dot{q}_k$, and y_{qk} to give:

$$a_{ik}\,|q_k|+b_{ik}\,|\dot{q}_k| \leq \chi_{ik}\sqrt{q_k^{\,2}+\dot{q}_k^{\,2}} = \chi_{ik}\,\|y_{qk}\|.$$

Substituting this into Eqn.(5-3-26) results in:

$$\begin{aligned}|\eta_i| &\leq \sum_{k=1}^{n} (a_{ik}|q_k|+b_{ik}|\dot{q}_k|)+c_{ik} \\ &\leq \sum_{k=1}^{n} \chi_{ik}\sqrt{q_k^{\,2}+\dot{q}_k^{\,2}}+c_{ik} \\ &= \sum_{k=1}^{n} \chi_{ik}\,\|y_{qk}\| + c_{ik}. \end{aligned} \tag{5-4-11}$$

which is Eqn.(5-4-10).

It is easy to show that, by this state realization, the transfer function of state equation Eqns.(5-4-9) to (5-4-13) is

$$w_i(s)=h_i^T(sI-A_i)^{-1}b_i=\frac{s+\alpha_i}{s^2+k_{vi}s+k_{pi}}.$$

It is also known that if the condition

$$\alpha_i \neq \frac{k_{vi}\pm\sqrt{k_{vi}^{\,2}-4k_{pi}}}{2} \tag{5-4-12}$$

is satisfied, (A_i,b_i) is a controllable pair, as under this condition

$$\begin{aligned}&\text{rank}\,[\,b_i \;\; A_ib_i] \\ &= \text{rank}\begin{bmatrix} 1 & \alpha_i-k_{vi} \\ \alpha_i-k_{vi} & -k_{pi}-k_{vi}(\alpha_i-k_{vi}) \end{bmatrix} \\ &= 2\end{aligned}$$

so that the controllability matrix [b_i A_ib_i] is of full rank.

Since k_{vi} and k_{pi} are all greater than zero and $L_i(s)$ is a Hurwitz polynomial with one zero, $w_i(s)$ in Eqn.(5-4-3) will be a strictly positive real transfer function if

$$k_{vi} > \alpha_i > 0, \tag{5-4-13}$$

because, according to the definition of a strictly positive real transfer function, by this condition the real part of the frequency response function is positive, i.e.,

$$\mathrm{Re}[w_i(j\omega)] = \frac{\alpha_i k_{pi} + (k_{vi}-\alpha_i)\omega^2}{(k_{pi}-\omega^2)^2 + (k_{vi}\omega)^2} \geq 0, \tag{5-4-14}$$

for all real ω. It should be noted that Eqns.(5-4-12) and (5-4-13) are further restrictions for k_{pi} , k_{vi} and α_i, in addition to the requirements given in the non-adaptive control law Eqn.(3-3-1b) and linear filter operator Eqn.(5-3-11) respectively.

As the error system transfer function Eqn.(5-4-3) is strictly positive real with condition Eqn.(5-4-12) and Eqn.(5-4-13), then according to the Kalman-Yacubovitch lemma [33][64], for triple [A_i, b_i, h_i] shown in Eqns.(5-4-9) – (5-4-13), there exist positive definite matrices P_i and real vectors v_i such that the following Lyapunov equation

$$A_i^T P_i + P_i A_i = -v_i v_i^T - \kappa_i U_i \tag{5-4-15}$$

and

$$P_i b_i = h_i \tag{5-4-16}$$

are satisfied, where $U_i = U_i^T > 0$ is a given positive definite matrix and $\kappa_i > 0$ a given constant which is small enough.

5-4-2. Algorithm and Stability

Based on previous discussions, in this section the adaptive control algorithm – the update law of $\hat{\theta}_i$ will be presented.

For error state equation Eqns.(5-4-7) and (5-4-8), an adaptation update law for the controller parameters vector $\hat{\theta}_i$ in Eqn.(5-3-14), is given as the following:

$$\dot{\hat{\theta}}_i = -\beta_i\Gamma_i\hat{\theta}_i - b_i^T P_i y_i \Gamma_i \delta_i \qquad \text{for } i=1,2,\ldots,n. \qquad (5\text{-}4\text{-}18a)$$

$$\beta_i = \begin{cases} \beta_{oi} & \text{if } \|\hat{\theta}_i\| > \hat{\theta}_{oi} \\ 0 & \text{if } \|\hat{\theta}_i\| \leq \hat{\theta}_{oi} \end{cases} \qquad (5\text{-}4\text{-}18b)$$

where β_i, β_{oi}, and $\hat{\theta}_{oi}$ are positive constants, $\Gamma_i=\Gamma_i^T>0$ is a positive definite matrix, $P_i^T=P_i>0$ is a solution of the Lyapunov equations $A_i^TP_i+P_iA_i=-v_iv_i^T-\kappa_iU_i$ and $P_ib_i=h_i$ is given by Eqn.(5-4-15) and Eqn.(5-4-16).

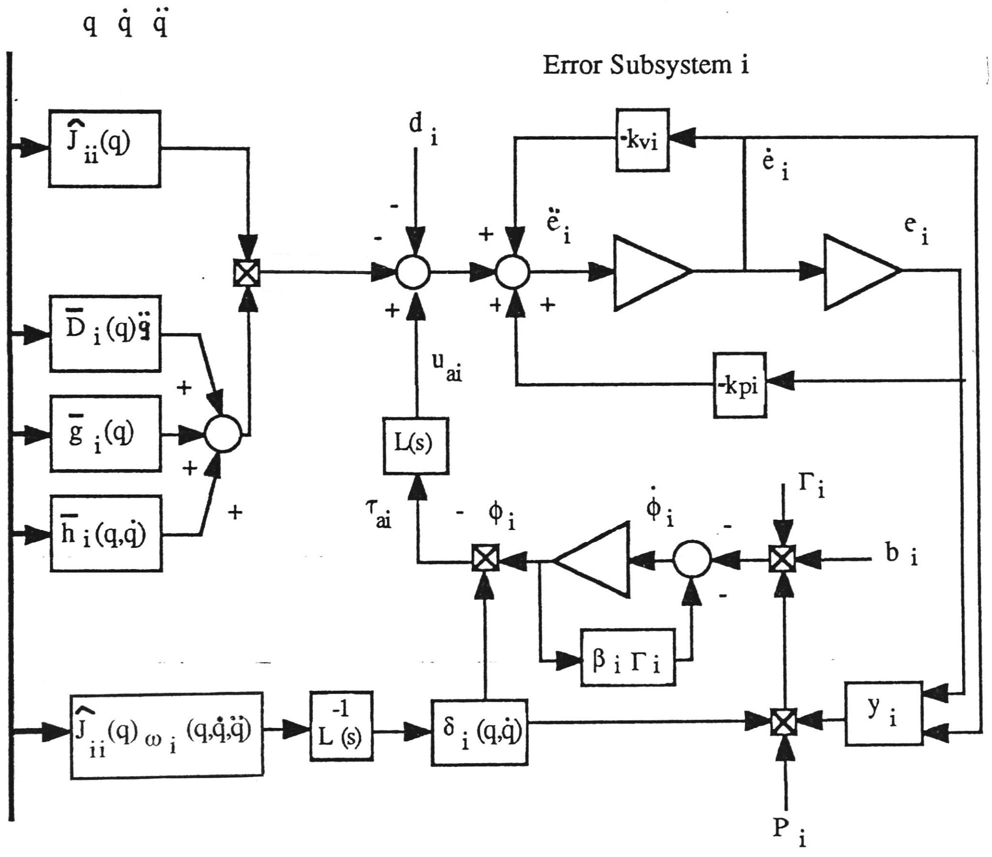

Fig.5-1. The architecture of closed loop error subsystem i with adaptive control component.

By means of this adaptive feedback control law, the resultant closed-loop structure of error subsystem i is given by the diagram in Fig.5-1 (ref. error equation Eqn.(5-3-10), adaptive control torque Eqn.(5-3-14) and parameter estimation up-date law Eqn.(5-4-18a and b). The bold line, in the left side of the diagram, represents the global feedback signals of the positions, velocities and accelerations of the overall system shared by subsystem 1, 2, ..., n. There are three input components to the error subsystem i. One is the dominant feedback

$$\bar{\theta}_i^T L_i^{-1}(s)\hat{J}_{ii}(q)\omega_i(q,\dot{q},\ddot{q}) = L_i^{-1}(s)\hat{J}_{ii}(q)[\bar{D}(q)_i\ddot{q}+\bar{h}_i(q,\dot{q})+\bar{g}_i(q)],$$

where $\bar{D}(q)_i$ is the i-th row of $\bar{D}(q)$: another is the interconnection d_i; and the last is the adaptive control torque u_{ai}. This torque is given by substituting Eqn.(5-4-18a and b) into Eqn.(5-3-14). In accordance with $\phi_i=\hat{\theta}_i-\theta_i$ and θ_i = constant, it follows that $\dot{\phi}_i = \dot{\hat{\theta}}_i$, that is, $\dot{\phi}_i$ has the same form as $\dot{\hat{\theta}}_i$ as shown in Fig.5-1.

The bottom of the diagram shows the adaptive control mechanism. Comparing this diagram with the error subsystem configuration in Fig.4-1, it can be seen that in this new structure the operators $L_i^{-1}(s)$ following the error system states in Fig.4-1 have been removed so this system is simpler than that in Chapter 4. This means that the error system states (position and velocity errors e_i and $\dot{e}_i$) can be observed directly. It will be seen in the following that the boundedness of e_i and $\dot{e}_i$ can be obtained.

For the error systems and this adaptive control law, the following theorem establishes the boundedness of the tracking error states and parameter estimation errors as well as their convergence rates:

Theorem 5-1:

Denote

$$\sigma_i = \frac{1}{2}\kappa_i \min\lambda(U_i), \qquad \text{for } i=1, 2,\ldots, n, \tag{5-4-19}$$

where κ_i and U_i are given by Eqn.(5-4-15), and $\min\lambda(U_i)$ is the minimum eigenvalue of U_i. Let

$$r_{ij} = (\chi_{ij} + \frac{c_{ij}}{\|y_{dj}\|})\|P_i b_i\|, \qquad \text{for } i, j=1, 2,\ldots, n. \tag{5-4-20}$$

where $P_i b_i$, and χ_{ij} are given by Eqns.(5-4-16) and Eqn.(5-4-10) respectively.

Suppose there exists a vector $\pi=[\pi_1, \pi_2,\ldots, \pi_n]^T$ with $\pi_i \geq 0$ for $i=1,2,\ldots,n$, such that the matrix defined by

$$M = \begin{bmatrix} \sigma_1-2r_{11} & -\pi_1 r_{12}-\pi_2 r_{21} & \cdots & -\pi_1 r_{1n}-\pi_n r_{n1} \\ -\pi_2 r_{21}-\pi_1 r_{12} & \sigma_2-2r_{22} & \cdots & -\pi_2 r_{2n}-\pi_n r_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ -\pi_n r_{n1}-\pi_1 r_{1n} & \cdots & \cdots & \sigma_n-2r_{nn} \end{bmatrix} > 0, \tag{5-4-21}$$

i.e., M is positive definite, then:

i). The tracking error y_i given by Eqn.(5-4-7) and the parameter estimate errors ϕ_i caused by the estimation law Eqn.(5-4-18) are uniformly bounded, for $i=1,2,\ldots,n$;

ii). There exist positive constants b_o and ζ_o such that the overall system tracking error $y=[y^T_1, y^T_2, \ldots, y^T_n]^T$ and the parameter estimation errors $\phi=[\phi^T_1, \phi^T_2, \ldots, \phi^T_n]^T$ converge to the residual set

$$D = \{ y, \phi \mid \|y\|^2 + \|\phi\|^2 \leq \frac{2}{b_o\zeta_o}\bar{K} \} \tag{5-4-22}$$

with a rate at least as fast as $e^{-\overline{b_o t}}$. In Eqn.(5-4-22) $\bar{K}$ is given by

$$\bar{K}=\frac{1}{2}\sum_{i=1}^{n}\pi_i\left[\,\beta_{oi}(\|\tilde{\theta}_i\|+\hat{\theta}_{oi})^2+\beta_{oi}\|\tilde{\theta}_i\|^2+\frac{1}{\sigma_i}y_{oi}^2\,\right], \tag{5-4-23}$$

where

$$y_{oi}=\sup_t\sum_{j=1}^{n} r_{ij}\|y_{dj}(t)\|, \tag{5-4-24}$$

and $\beta_{oi}>0$ is a design parameter.

The proof of Theorem 5-1 is given in Appendix C.

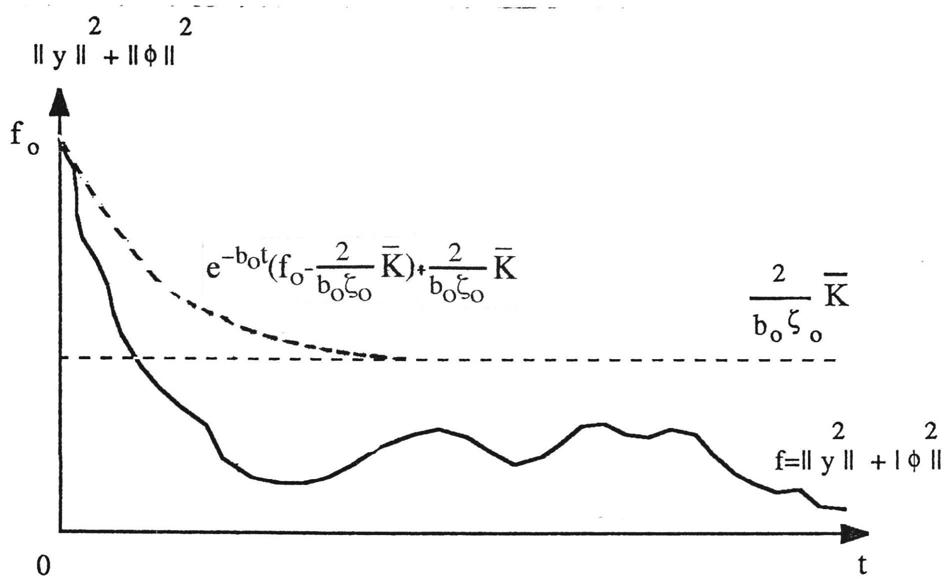

Fig. 5-2. The converge ratio and the boundedness of $\|y\|^2+\|\phi\|^2$.

The convergence property and the boundedness of $\|y\|^2+\|\phi\|^2$ are illustrated in Fig.5-2. In this figure, the area below $\frac{2}{b_o\zeta_o}\bar{K}$ is the region satisfying

$$\|y\|^2+\|\phi\|^2\leq\frac{2}{b_o\zeta_o}\bar{K}\,.$$

Theorem 5-1 declares that under condition Eqn.(5-4-21), scalar function $\|y\|^2+\|\phi\|^2$ will converge to a region satisfying $\|y\|^2+\|\phi\|^2\leq\frac{2}{b_o\zeta_o}\bar{K}$ from its initial value with a rate faster than exponentical attenuation $e^{-b_o t}$. Geometrically, this means function $f=\|y\|^2+\|\phi\|^2$ will decline from its initial value along a trajectory below the function $e^{-b_o t}(f_o - \frac{2}{b_o \zeta_o}\bar{K}) + \frac{2}{b_o \zeta_o}\bar{K}$ in the interval $[0,\infty)$. It should be noted that as:

$$\bar{K} = \frac{1}{2}\sum_{i=1}^{n} \pi_i \left[\, \beta_{oi}(\|\tilde{\theta}_i\| + \hat{\theta}_{oi})^2 + \beta_{oi}\|\tilde{\theta}_i\|^2 + \frac{1}{\sigma_i} y_{oi}^2 \,\right],$$

in which y_{oi} is a constant proportional to the magnitudes of interconnections and the supremum of reference trajectory y_{di} (see Eqns.(5-4-20) and (5-4-24)), the size of $\frac{2}{b_o \zeta_o}\bar{K}$ will be proportional to the strengths of the interconnections and the supremums of the reference trajectories.

A further geometrical explanation of Theorem 5-1 is given in Fig.5-3. The Lyapunov function of the overall system $v(y, \phi)$ given by Eqn.(AC-1) in Appendix C can be interpreted as a super ellipsoid defined above the super plane spanned by y, ϕ. For a bounded initial condition $v(y(0), \phi(0))$, the scalar function $v(y(t), \phi(t))$ will decline along the solution trajectory of state equation Eqn.(5-4-7) with a rate at least as fast as exponential function $e^{-b_o t}$. In this figure, the system state trajectory $(y(t), \phi(t))$, driven by the adaptive control law, moves along the projection of $v(y, \phi)$ on the (y, ϕ) super plane and enters into the residual set D given by the shadowed area in the super plane. The residual set D is formed by a projection of the ellipsoid surface below the contour line $\|y\|^2 + \|\phi\|^2 \leq \frac{2}{b_o \zeta_o}\bar{K}$.

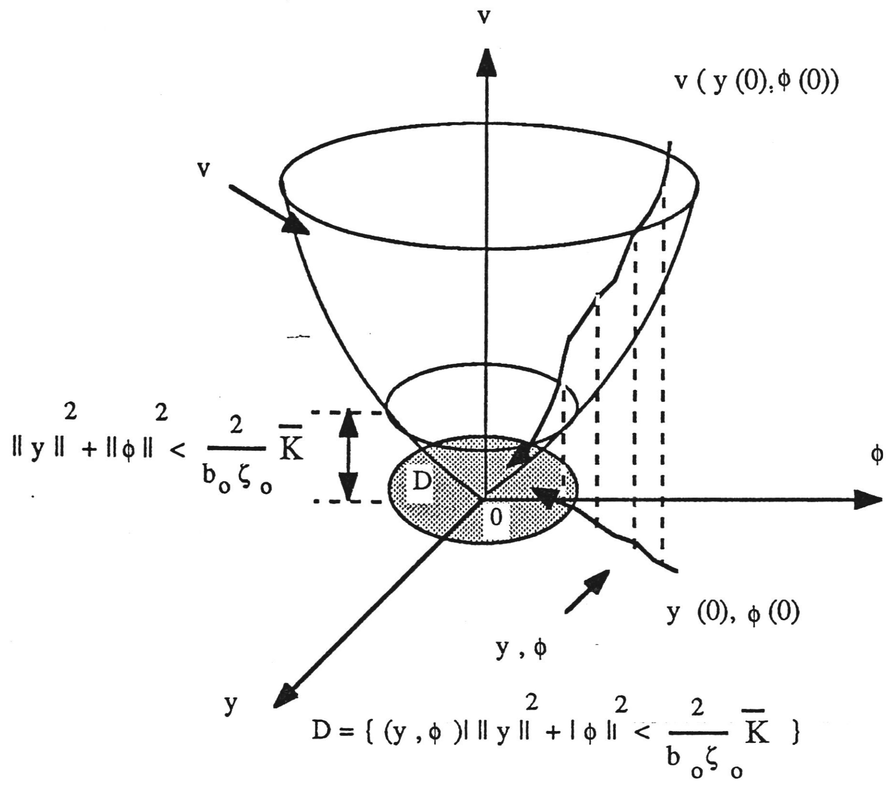

Fig.5-3. The geometrical interpretation of the convergences of the tracking errors and parameter estimation errors.

5-4-3. Bounded Control Torque u_{ai}

The final implementation of adaptive control torque is obtained by substituting Eqn.(5-4-18a) into Eqn.(5-3-14) which gives

$$u_{ai}=(s+\alpha_i)\hat{\theta}_i^T\delta_i(q,\dot{q}).$$

Denoting $\mu_i(q,\dot{q})=\hat{\theta}_i^T\delta_i(q,\dot{q})$, this can be expressed as

$$\begin{aligned} u_{ai} &= (s+\alpha_i)\mu_i(q,\dot{q}) \\ &= \dot{\mu}(q,\dot{q})+\alpha_i\mu_i(q,\dot{q}), \end{aligned} \tag{5-4-25}$$

which is a proportional-plus-differential control law. It should be noted that theoretically the control law Eqn.(5-4-25) contains information about accelerations since it can also be rewritten as

$$u_{ai}=\dot{\hat{\theta}}_i^T\delta_i(q,\dot{q})+\hat{\theta}_i^T\dot{\delta}_i(q,\dot{q})+\alpha_i\hat{\theta}_i^T\delta_i(q,\dot{q})$$

$$=\dot{\hat{\theta}}_i^T\delta_i(q,\dot{q})+\hat{\theta}_i^T(s+\alpha_i)\delta_i(q,\dot{q}).$$

$$=\dot{\hat{\theta}}_i^T\delta_i(q,\dot{q})+\hat{\theta}_i^T\hat{J}_{ii}(q)\omega_i(q,\dot{q},\ddot{q}). \qquad (5\text{-}4\text{-}26)$$

in which the accelerations are involved explicitly. As the derivative of $\mu_i(q,\dot{q})$ is involved in Eqn.(5-4-25), there may be concern about the boundedness of $\dot{u}_{ai}$. However, the following theorem ensures the bounded magnitude of u_{ai}:

Theorem 5-2:

For the system Eqn.(5-4-7) and Eqn.(5-4-8), and parameter update law Eqn.(5-4-18), control law Eqn.(5-4-25) (which is equivalent to Eqn.(5-3-14)) is bounded.

Proof:

As control law Eqn.(5-4-25) is equivalent to Eqn.(5-4-26), if it can be proved that $\hat{\theta}_i$, $\dot{\hat{\theta}}_i$, $\hat{J}_{ii}(q)$ and $\omega_i(q,\dot{q},\ddot{q})$ are all bounded then u_{ai} must be bounded. According to Assumption 3-3), $\hat{D}(q)$ is positive definite, which implies $\bar{\hat{J}}_{ii}(q)$ is bounded. From Theorem 5-1, it is known that both e_i and $\dot{e}_i$ are bounded as y_i is bounded for i=1,2,...,n. This results in bounded $\delta_i(q,\dot{q})$ as the trajectory signals q_d and $\dot{q}_d$ are both bounded. In view of Eqn.(5-3-15), bounded e_i, $\dot{e}_i$, $\delta_i(q,\dot{q})$ and ϕ_i lead to bounded $\ddot{e}$ for i=1,2,...,n, which means $\ddot{q}$ is bounded as well since $\ddot{q}_d$ is bounded. Then it follows that $\omega_i(q,\dot{q},\ddot{q})$ is also bounded according to Assumption 3-1). Furthermore, from Theorem 5-1, the bounded ϕ_i implies that $\hat{\theta}_i = \theta_i - \phi_i$ is bounded as θ_i is a constant vector. By the control law Eqn.(5-3-14), it can be seen that $\dot{\phi}_i$ is bounded as ϕ_i, y_i and δ_i are all bounded which in return implies that $\dot{\hat{\theta}}_i$ is bounded. Then Theorem 5-2 follows.

5-5. COMMENTS

In Theorem 5-1, as pointed out in [27], the positive definiteness of the matrix M (see Eqn.(5-4-21)) is a sufficient condition to ensure stable decentralized systems even in the cases of non-adaptive control. From Corollary 5-1, Eqns.(5-4-20) can be written as

$$r_{ij} = (\chi_{ij} + \frac{c_{ij}}{\|y_{dj}\|})\|P_i b_i\| \geq (\sqrt{a_{ij}^2 + b_{ij}^2} + \frac{c_{ij}}{\|y_{dj}\|})\|P_i b_i\|, \qquad \text{for } i, j = 1, 2, \ldots, n.$$

Since a_{ij}, b_{ij} and c_{ij} are all proportional to the magnitudes of interconnections (see Eqn.(5-3-26)), smaller r_{ij} will mean smaller interconnections. The satisfaction of this condition implies that when the interconnections among the subsystems are sufficiently weak, as in the case when r_{ij} are all sufficiently small, matrix M will become M=diag$\{\sigma_i\}$ which is positive definite, as $\sigma_i>0$ for i=1, 2, ..., n.

In adaptive control, as the dynamic parameters of the controlled system are unknown for a given robot, it is difficult to check if the condition M>0 is satisfied since r_{ij} depend on the system's true parameters. However, it is possible to check this condition by experiments and system analysis to find out the approximate boundedness of these parameters. It is worth noting that if the robot has a non-direct drive arm and the gear ratios of the transmissions are relatively high, the interconnection torques among subsystems will be reduced dramatically at the motor shafts as the inertia of the robot links will be reduced by the square of the gear ratio. For these robots the interconnections are likely to be sufficiently weak.

From Eqns.(5-4-22) and (5-4-23), both repeated here

$$D = \{\ y, \phi \mid \|y\|^2 + \|\phi\|^2 \le \frac{2}{b_o\zeta_o}\ \bar{K}\ \} \tag{5-5-1a}$$

$$\bar{K} = \frac{1}{2}\sum_{i=1}^{n}\pi_i\,[\ \beta_{oi}(\|\theta_i\|+\hat{\theta}_{oi})^2+\beta_{oi}\|\theta_i\|^2+\frac{1}{\sigma_i}\,y_{oi}^{\ 2}\ ], \tag{5-5-1b}$$

it can be seen that the size of the residual set D is proportional to y_{oi}. According to Eqns.(5-4-20) and (5-4-24)

$$y_{oi} = \sup_t \sum_{j=1}^{n} r_{ij}\,\|y_{dj}(t)\| = \sup_t \sum_{j=1}^{n} (\chi_{ij}\,\|y_{dj}\|+c_{ij})\ \|P_i b_i\|$$

where χ_{ij} and c_{ij} are proportional to the magnitudes of the interconnections as mentioned before, and $y_{dj}=[q_{dj}\ \dot{q}_{dj}]^T$ are the reference trajectories. This is physically understandable as stronger interconnections or disturbances and faster trajectories will lead to a bigger residual set and therefore larger tracking errors.

In view of Eqn.(5-5-1), the size of D is also influenced by design parameters β_{oi}. If β_{oi} are large, from Eqn.(AC-9) in Appendix C, it follows that

$$b_o = \min_i\,(\frac{\lambda_m}{\pi_i p_i}) \le \min_i\,(\frac{\beta_{oi}}{\gamma_i}),$$

and D is determined by

$$\|y\|^2+\|\phi\|^2 \le \frac{2}{\min_i\,(\frac{\lambda_m}{\pi_i p_i})\zeta_o}\ \bar{K}. \tag{5-5-2}$$

If β_{oi} are small, then

$$b_o = \min_i\,(\frac{\beta_{oi}}{\gamma_i}) \le \min_i\,(\frac{\lambda_m}{\pi_i p_i}),$$

and residual set D is determined by

$$\|y\|^2+\|\phi\|^2 \le \frac{2}{\min_i\,(\frac{\beta_{oi}}{\gamma_i})\zeta_o}\ \bar{K}.$$

As b_o is proportional to the convergence rate of the tracking errors and estimation errors, for small β_{oi}, the convergence rate will decrease and D will be small. However, if β_{oi} is large enough the convergence rate will depend on $\min_i (\frac{\lambda_m}{\pi_i p_i})$ only and the residual set will increase according to Eqns.(5-5-1) and (5-5-2).

As mentioned in Section 4-3, unlike cases involving some plants in which the system states may not be observable, in robot motion control the system states such as positions and velocities of each joints are always measurable. It is possible to define the reference trajectories such that $q_d(0)=q(0)$ and $\dot{q}_d(0)=\dot{q}(0)$, i.e., $e(0)=\dot{e}(0)=0$. In this case, the state trajectory (y, ϕ) will start from $y(0)=[e^T(0)\ \dot{e}^T(0)]^T=0$ in (y, ϕ) plane and remain as shown in the area between the dashed lines in the diagram shown in Fig.5-4.

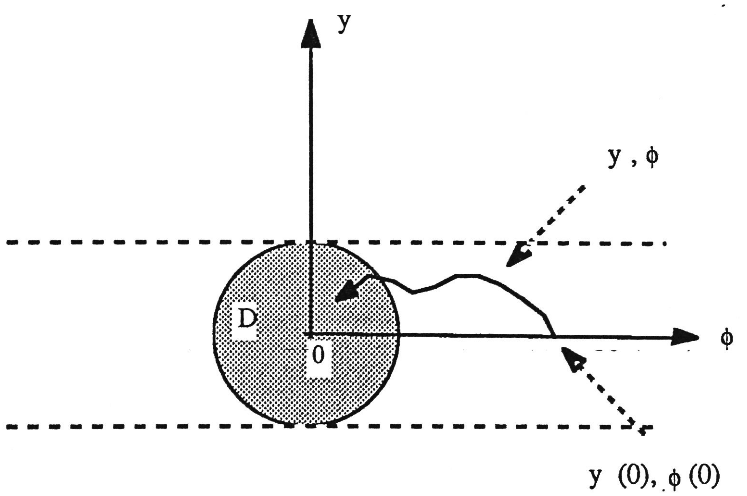

Fig. 5-4. The state trajectory of (y,ϕ) with initial condition tracking error $y(0)=0$.

Theorem 5-1 establishes the boundedness of the parameter errors ϕ without $\delta_i(q,\dot{q})$ being persistently excited [1][47]. Due to the interconnections between the subsystems the parameter estimation error vector ϕ is not able to converge to its true value, i.e., the unbiased estimates cannot be obtained.

It is worth noting that in Chapter 4 and this chapter, the linear operators $L_i^{-1}(s)$ were introduced to operate on d_i. For instance, in this chapter Eqn.(5-3-15) can be written as

$$\ddot{e}_i+k_{vi}\dot{e}_i+k_{pi}e_i=L_i(s)[\phi_i^T\delta_i(q,\dot{q})-\eta_i]$$

$$=L_i(s)\phi_i^T\delta_i(q,\dot{q})-L_i(s)L_i^{-1}(s)d_i, \qquad (5\text{-}5\text{-}3)$$

according to Eqn.(5-3-16):

$$\eta_i=L_i^{-1}(s)d_i. \qquad (5\text{-}5\text{-}4)$$

This treatment is only for theoretical analysis and Eqn.(5-5-4) is neither required nor possible to be implemented as d_i is not measurable. Even though it has not been claimed that d_i is bounded so far, it is easily proved that this is true. In accordance with Theorem 5-1, it is known that ϕ, e, $\dot{e}$ are all bounded and therefore q and $\dot{q}$ are bounded as well. From Theorem 5-2, u_a is bounded. In view of Eqn.(5-5-3) it can be seen that $\ddot{e}_i$ are bounded for i=1,2, ..., n, which implies that $\ddot{q}_i$ are all bounded for i=1,2, ..., n, according to Assumption 2-2) in Section 2-3-2. Thus in view of Eqn.(5-3-9) d_i is bounded.

5-6. SUMMARY

This chapter presented a major contribution of this thesis on the trajectory following control of robotic manipulators. The main results are a robust adaptive control algorithm based on the decentralized error system structure. A theoretical analysis of the stability and convergence of the overall closed-loop system is also presented.

Using the "linear in parameter" description, the unknown dynamic parameters appear as a constant vector in the error subsystems obtained in Chapter 3. For this constant vector, the linear operators $P_L(\theta)$ proposed by [48] are introduced. As a result of this, the acceleration measurements of the system state can be avoided by using the inverse operators $L_i^{-1}(s)$ and an additional zero is obtained by the positive operator $L_i(s)$. As the operators are chosen in such a way that the strictly positive real condition is satisfied for each subsystem, the Lyapunov direct method can be used in controller design.

The boundedness of the magnitudes of the filtered interconnections are investigated in Section 5-3-2 and the conclusion that the interconnections are all bounded if the positions and velocities of the overall system are bounded is obtained. This is of substantial importance in the sense that under this condition the overall system appears as a standard decentralized system to which the adaptive control scheme given by [27] can be applied. Another advantage of this is that the conditions (C-3) to (C-6), which are required in the scheme proposed in Chapter 4, can be removed in the stability and convergence proofs.

As the error system is strictly positive real, the Kalman-Yacubovitch lemma is used in controller design based on the same idea used in [27]. The final theoretical results, presented by Theorem 5-1, are the quantitative boundedness for both position and velocity tracking errors and parameter estimation errors and their convergence rate to a residual set inside the given bounds. Moreover, the geometrical interpretation of this theorem is presented and some comments on features of the scheme and the determinations of controller design parameters are given. It also has been proved that the magnitude of final implementation of the adaptive control torque is bounded as well.

This algorithm maintains the virtues of the algorithm proposed in Chapter 4. Only the diagonal elements of the inverse of inertia estimation matrix are required. This inverse always exists because of the two component control torque configuration. In addition no acceleration measurement is needed. Other advantages include: the error system states are the position and velocity errors directly instead of filtered ones in the former scheme: the relationship between the boundedness of interconnections and that of the overall system position and velocities were found so that the the theoretical analysis is based on a more realistic foundation: and the conditions (C-3) to (C-6) required by the method in Chapter

4 are removed. In this case, as soon as the positive M matrix exists, the convergence of tracking error and parameter error to the residual set with a quantitative boundedness is guaranteed.

The practical effect of requiring M to be positive definite is that the couplings between subsystems are not very strong. This algorithm also introduces a dead zone not required by the first.

Chapter 6.

A CASE STUDY AND SIMULATIONS

6-1. INTRODUCTION

In this chapter, to illustrate the adaptive control schemes proposed in Chapters 4 and 5, a case study on controller design for an industrial manipulator system will be presented. The model used is based on the equations of motion of a SCARA robot manipulator. For this manipulator, the controller design procedures and the determinations of adaptive controller parameters using the schemes in Chapters 4 and 5 will be shown. After this, there will be a presentation of some numerical simulation results of this robot's trajectory following performances under the control of the controllers obtained in the case study. In order to evaluate the performance of these control schemes, the computed torque schemes are also implemented in the simulations to compare with the results of the proposed schemes.

The chapter is organized as follows. In Section 6-2, the dynamic model of the robot used will be introduced. For this two joint SCARA robot arm, some issues related to the implementation of proposed adaptive control algorithms will be investigated as well as the computed torque approach in Section 6-3. In Section 6-4, the determination of adaptive controller design parameters will be discussed, as the correct choice of these parameters is very important in getting the desired performance for the controlled robots. In Section 6-5 two types of trajectories used in the simulations will be stated and then the simulation results will be presented together with some comments. Finally, Section 6-6 is a summary .

6-2. ROBOT DYNAMIC MODEL

The robot model applied here is the same one used in the example in Section 2-2-3 which is a dynamic model of the first two revolute joints of a SCARA robotic manipulator. The system configuration is shown in Fig. 6-1. Without loss of generality, the motion equations of this robot are derived under the following simplifying assumptions:

(1). Both links have a uniform, square cross section and each link is divided along its length into an infinite number of these square cross sections;

(2). The mass of each cross sectional segment is concentrated at a point in the centre of the cross section so that the link's mass can be thought of as being evenly distributed along its long axis of symmetry;

(3). The payload is grasped firmly by the robot end-effector with its mass concentrated at the free end of link 2 so that it can be regarded as a point mass;

(4). The friction forces in all joints and the dynamics of all actuators are ignored.

As a result of this, the dynamic equations obtained are totally determined by physical parameters such as mass and geometrical size of the robot links and joint variables such as positions, velocities and accelerations. For this particular robot, since both links are moving horizontally, the gravitational energy function is a constant, i.e. P(q)=constant, which results in a zero vector of gravitational torque, i.e., $g(q)=\partial P(q)/\partial q=0$.

Using the Lagrange equations, the dynamic equation Eqn.(2-2-7) of this robot then becomes:

$$D(q)\ddot{q}+h(q,\dot{q})=u. \tag{6-2-1}$$

In Eqn.(6-2-1), D(q) and $h(q,\dot{q})$ are given by

$$D(q)=\begin{bmatrix} d_{11}(q) & d_{12}(q) \\ d_{12}(q) & d_{22}(q) \end{bmatrix}$$

$$h(q,\dot{q})=\begin{bmatrix} h_1(q,\dot{q}) \\ h_2(q,\dot{q}) \end{bmatrix}$$

where

$$d_{11}(q) = c_1+c_3+c_4m_L+(c_2+2L_1L_2m_L)\cos q_2 \tag{6-2-2a}$$

$$d_{12}(q) = d_{21}(q) = c_3+L_2{}^2m_L+(c_2/2+L_1L_2m_L)\cos q_2 \tag{6-2-2b}$$

$$d_{22}(q) = c_3+L_2{}^2m_L \tag{6-2-2c}$$

and

$$h_1(q,\dot{q})=-(c_2/2+L_1L_2m_L)\sin q_2(2\,\dot{q}_1+\dot{q}_2)\dot{q}_2, \tag{6-2-3a}$$

$$h_2(q,\dot{q})=(c_2/2+L_1L_2m_L)\sin q_2\,\dot{q}_1\dot{q}_1. \tag{6-2-3b}$$

On the other hand, dynamics can also be shown by Eqn.(2-2-5b)

$$\frac{d}{dt}(D(q)\dot{q})+s(q,\dot{q})=u. \tag{6-2-4}$$

in which

$$s(q,\dot{q})=\begin{bmatrix} s_1(q,\dot{q}) \\ s_2(q,\dot{q}) \end{bmatrix}$$

with

$$s_1(q,\dot{q})=0, \tag{6-2-5a}$$

$$s_2(q,\dot{q})=(c_2/2+L_1L_2m_L)\sin q_2(\dot{q}_1\dot{q}_1+\dot{q}_1\dot{q}_2). \tag{6-2-5b}$$

In the equations above

$$c_1 = L_1^2(m_1/3+m_2) \tag{6-2-6a}$$

$$c_2 = L_1L_2m_2 \tag{6-2-6b}$$

$$c_3 = L_2^2m_2/3 \tag{6-2-6c}$$

$$c_4 = L_1^2+L_2^2 \tag{6-2-6d}$$

where L_i (m) and m_i (kg) are the lengths and the masses of link i (i=1, 2.) and m_L(kg) is the mass of payload fixed at the end of link 2;

It should be noticed that by neglecting the friction forces in the system model the dynamic equations are made simpler, but the resultant non-friction dynamics are harder to control as all damping forces are zero. This model corresponds more closely to a direct drive manipulator.

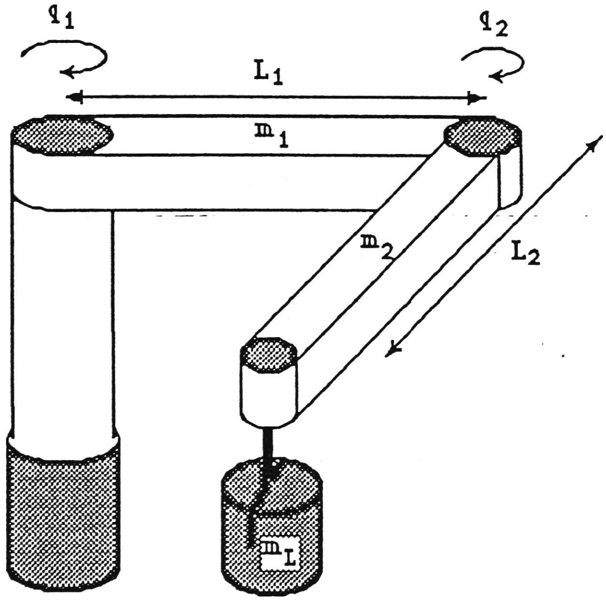

Fig. 6-1. The architecture of the SCARA robot used in the study and simulations.

6-3. CONTROL ALGORITHM IMPLEMENTATIONS

In order to simulate realistic cases of model-based controller designs, in which the true values of system parameters L_i, m_i together with the payload m_L are not known exactly, it is assumed that there are some a priori estimates of these values available. The estimates of L_i, m_i, and m_L are expressed by $\hat{L}_i$, $\hat{m}_i$ and $\hat{m}_L$ respectively (i=1 and 2). In this section, the controller design procedures will be shown using this set of estimates. Based on these estimates, the adaptive control system structures proposed in Chapters 4 and 5 are presented. The computed torque scheme is also reviewed since the adaptive control schemes proposed utilize this scheme and simulation results of the pure computed torque controller will be presented in Section 6-5 to compare with the performances of adaptive control laws proposed by this thesis.

6-3-1. Computed Torque Approach

Using $\hat{L}_i$, $\hat{m}_i$ and $\hat{m}_L$, the estimate of inertial matrix becomes

$$\hat{D}(q) = \begin{bmatrix} \hat{d}_{11}(q) & \hat{d}_{12}(q) \\ \hat{d}_{12}(q) & \hat{d}_{22}(q) \end{bmatrix} \tag{6-3-1}$$

As mentioned in Chapter 3, it is always possible to choose $\hat{L}_i$, $\hat{m}_i$ and $\hat{m}_L$ in such a way that the resulting estimate matrix $\hat{D}(q)$ is positive definite. The elements of $\hat{D}(q)$ are given by

$$\hat{d}_{11}(q)= \hat{c}_1+\hat{c}_3+\hat{c}_4\hat{m}_L+(\hat{c}_2+2\hat{L}_1\hat{L}_2\hat{m}_L)\cos q_2 \tag{6-3-2a}$$

$$\hat{d}_{12}(q) =\hat{c}_3+\hat{L}_2{}^2\hat{m}_L+(\hat{c}_2/2+\hat{L}_1\hat{L}_2\hat{m}_L)\cos q_2 \tag{6-3-2b}$$

$$\hat{d}_{22}(q) =\hat{c}_3+\hat{L}_2{}^2\hat{m}_L \tag{6-3-2c}$$

where $\hat{c}_i$ are the estimates of c_i's, which are obtained by substituting $\hat{L}_i$, $\hat{m}_i$ and $\hat{m}_L$ into the right hand sides of Eqn.(6-2-6).

At the same time the estimated centrifugal and Coriolis torque vector is

$$\hat{h}(q,\dot{q})=\begin{bmatrix}\hat{h}_1(q,\dot{q})\\ \hat{h}_2(q,\dot{q})\end{bmatrix}$$

with

$$\hat{h}_1(q,\dot{q})=-\hat{L}_1\hat{L}_2(\hat{m}_2/2+\hat{m}_L)\sin q_2\dot{q}_2(2\dot{q}_1-\dot{q}_2),$$

$$\hat{h}_2(q,\dot{q})=\hat{L}_1\hat{L}_2(\hat{m}_2/2+\hat{m}_L)\sin q_2\dot{q}_1\dot{q}_2.$$

Using $\hat{D}(q)$ and $\hat{h}(q,\dot{q})$, the computed torque control law Eqn.(3-3-1b) becomes

$$u_1=\hat{D}(q)[\ddot{q}_d+K_v(\dot{q}_d-\dot{q})+K_p(q_d-q)]+\hat{h}(q,\dot{q}) \tag{6-3-3}$$

where

$$K_v=\begin{bmatrix}k_{v1} & 0\\ 0 & k_{v2}\end{bmatrix} \tag{6-3-4a}$$

$$K_p=\begin{bmatrix}k_{p1} & 0\\ 0 & k_{p2}\end{bmatrix} \tag{6-3-4b}$$

with $k_{v1}=k_{v2}=5$ and $k_{p1}=k_{p2}=25$. This set of parameters corresponds to the open loop characteristic equation of the error system

$$c_i(s)=s^2+2\zeta\omega_n s+\omega_n^2,$$

with damping ratio ζ=0.5 and undamped natural frequency ω_n=5 rad/s for sub system 1 and 2.

Placing Eqn.(6-3-3) into the right hand side of Eqn.(6-2-1) and noticing that u_a=0 in the case of pure computed torque control, the error dynamics are (see Eqn.(3-3-7)):

$$\ddot{e}+K_v\dot{e}+K_pe = \hat{J}_d(q)[(\bar{D}(q)\ddot{q})+\bar{h}(q,\dot{q})]+d. \tag{6-3-5}$$

in which

$$\hat{J}(q) = \frac{1}{\hat{d}_{11}(q)\hat{d}_{22}(q)-\hat{d}_{12}{}^2(q)}\begin{bmatrix} \hat{d}_{22}(q) & -\hat{d}_{12}(q) \\ -\hat{d}_{12}(q) & \hat{d}_{11}(q) \end{bmatrix}$$

$$\bar{D}(q) = \begin{bmatrix} d_{11}(q)-\hat{d}_{11}(q) & d_{12}(q)-\hat{d}_{12}(q) \\ d_{12}(q)-\hat{d}_{12}(q) & d_{22}(q)-\hat{d}_{22}(q) \end{bmatrix}$$

$$\bar{h}(q,\dot{q}) = \begin{bmatrix} h_1(q,\dot{q})-\hat{h}_1(q,\dot{q}) \\ h_2(q,\dot{q})-\hat{h}_2(q,\dot{q}) \end{bmatrix}$$

6-3-2. Adaptive Control

In this section, for this particular example, the adaptive controller designs will be illustrated. For brevity ADA1 and ADA2 are used as labels for the adaptive control algorithms proposed in Chapters 4 and 5 respectively.

In the adaptive control system structure ADA1, an additional component u_a is introduced into Eqn.(6-3-5), and this gives (see Eqn.(3-3-7))

$$\ddot{e}+K_v\dot{e}+K_pe = \hat{J}_d(q)[(\bar{D}(q)\ddot{q})+\bar{h}(q,\dot{q})]-u_a+d,$$

which is equivalent to Eqn.(3-3-10):

$$\ddot{e}+K_v\dot{e}+K_pe=\hat{J}_d(q)[\tfrac{d}{dt}(\bar{D}(q)\dot{q})+\bar{s}(q,\dot{q})]-u_a+d.$$

The i-th subsystem, corresponding to Eqn.(3-3-11) is:

$$\ddot{e}_i+k_{vi}\dot{e}_i+k_{pi}e_i=\hat{J}_{ii}(q)[\tfrac{d}{dt}(\bar{D}_i(q)\dot{q})+\bar{s}_i(q,\dot{q})]-u_{ai}+d_i$$

$$=\bar{\theta}_i^T\hat{J}_{ii}(q)\omega_i(q,\dot{q},\ddot{q})-u_{ai}+d_i,$$

for i=1 and 2.

In ADA2, as $u=u_1-u_2=u_1-\hat{D}(q)u_a$ and $\bar{D}(q)=D(q)-\hat{D}(q)$, $\bar{h}(q,\dot{q})=h(q,\dot{q})-\hat{h}(q,\dot{q})$ rather than $u=u_1+u_2=u_1+\hat{D}(q)u_a$ and $\bar{D}(q)=\hat{D}(q)-D(q)$, $\bar{h}(q,\dot{q})=\hat{h}(q,\dot{q})-h(q,\dot{q})$ in ADA1, it follows that

$$\ddot{e}+K_v\dot{e}+K_pe=\hat{J}_d(q)[(-\bar{D}(q)\ddot{q})-\bar{h}(q,\dot{q})]+u_a-d,$$

and the i-th subsystem, corresponding to Eqn.(3-3-11) is:

$$\ddot{e}_i+k_{vi}\dot{e}_i+k_{pi}e_i=\hat{J}_{ii}(q)[-\tfrac{d}{dt}(\bar{D}_i(q)\dot{q})-\bar{s}_i(q,\dot{q})]+u_{ai}-d_i$$

$$=-\bar{\theta}_i^T\hat{J}_{ii}(q)\omega_i(q,\dot{q},\ddot{q})+u_{ai}+d_i,$$

for i=1 and 2.

For this example, the diagonal elements of the inverse of the estimated inertia matrix are

$$\hat{J}_{11}(q)=\frac{\hat{d}_{22}(q)}{\hat{d}_{11}(q)\hat{d}_{22}(q)-\hat{d}_{12}^2(q)},$$

$$\hat{J}_{22}(q)=\frac{\hat{d}_{11}(q)}{\hat{d}_{11}(q)\hat{d}_{22}(q)-\hat{d}_{12}^2(q)},$$

where $\hat{d}_{ij}$ are given by Eqn.(6-3-1). The generalized known state vectors $\omega_1(q,\dot{q},\ddot{q})$ and $\omega_2(q,\dot{q},\ddot{q})$ in Eqn.(3-3-14) are

$$\omega_{11}^T(q,\dot{q},\ddot{q}) = \frac{d}{dt}[\,\dot{q}_1,\ \dot{q}_1\cos q_1,\ \dot{q}_2,\ \dot{q}_2\cos q_2],$$

$$\omega_{12}^T(q,\dot{q}) = 0,$$

$$\omega_{13}^T(q) = 0,$$

$$\omega_{21}^T(q,\dot{q},\ddot{q}) = \frac{d}{dt}[\,\dot{q}_1,\ \dot{q}_1\cos q_1,\ \dot{q}_2],$$

$$\omega_{22}^T(q,\dot{q}) = [\,\dot{q}_1\sin q_2(\dot{q}_1+\dot{q}_2)],$$

$$\omega_{23}^T(q) = 0$$

respectively.

The second, third and sixth equations above result from the fact that $\bar{s}_1(q,\dot{q})=0$ and $g(q)=0$. After feeding ω_i into the filter operator Eqn.(4-2-5), the filtered observation nonlinear vector $\delta_i(q,\dot{q})$ in Eqn.(4-2-8) will be

$$\delta_1^T(q,\dot{q}) = \frac{1}{s+\alpha_i}\,\hat{J}_{11}(q)\,\frac{d}{dt}[\,\dot{q}_1,\ \dot{q}_1\cos q_1,\ \dot{q}_2,\ \dot{q}_2\cos q_2],$$

$$\delta_2^T(q,\dot{q}) = \frac{1}{s+\alpha_i}\,\hat{J}_{22}(q)\,\frac{d}{dt}[\,\dot{q}_1,\ \dot{q}_1\cos q_1,\ \dot{q}_2,\ \dot{q}_1\sin q_2(\dot{q}_1+\dot{q}_2)].$$

The parameter error vector $\bar{\theta}_i$ are

$$\bar{\theta}_1^T=[\ \bar{\theta}_{11}\ \bar{\theta}_{12}\ \bar{\theta}_{13}\ \bar{\theta}_{14}\ ]$$

$$\bar{\theta}_2^T=[\ \bar{\theta}_{21}\ \bar{\theta}_{22}\ \bar{\theta}_{23}\ \bar{\theta}_{24}\ ]$$

and according to Eqns.(3-2-4a and c), (6-2-4) and (6-3-2) the elements in the above vectors are

$$\bar{\theta}_{11}=d_{111}-\hat{d}_{111}=c_1+c_3+c_4m_L-(\hat{c}_1+\hat{c}_3+\hat{c}_4\hat{m}_L)$$

$$\bar{\theta}_{12}=d_{112}-\hat{d}_{112}=c_2+2L_1L_2m_L-(\hat{c}_2+2\hat{L}_1\hat{L}_2\hat{m}_L)$$

$$\bar{\theta}_{13}=d_{121}-\hat{d}_{121}=c_3+L_2^2m_L-(\hat{c}_3+\hat{L}_2^2\hat{m}_L)$$

$$\bar{\theta}_{14}=d_{122}-\hat{d}_{122}=c_2/2+L_1L_2m_L-(\hat{c}_2/2+\hat{L}_1\hat{L}_2\hat{m}_L)$$

$$\bar{\theta}_{21}=d_{211}-d_{211}=c_3+L_2{}^2m_L-(\hat{c}_3+\hat{L}_2{}^2\hat{m}_L)$$

$$\bar{\theta}_{22}=d_{212}-\hat{d}_{212}=c_2+2L_1L_2m_L-(\hat{c}_2/2+\hat{L}_1\hat{L}_2\hat{m}_L)$$

$$\bar{\theta}_{23}=d_{221}-\hat{d}_{221}=c_3+L_2{}^2m_L-(\hat{c}_3+\hat{L}_2{}^2\hat{m}_L)$$

$$\bar{\theta}_{24}=k_{21}-\hat{k}_{21}=c_2/2+L_1L_2m_L-(\hat{c}_2/2+\hat{L}_1\hat{L}_2\hat{m}_L)$$

6-4. DETERMINATION OF ADAPTIVE CONTROLLER PARAMETERS

The performance of the adaptive control system is critically dependent on the correct setting of the controller parameters. In this section, the choice of the controller parameters in ADA1 and ADA2 will be stated.

6-4-1. Design Parameter Determinations for ADA1

First of all, the system matrices A_i are chosen as

$$A_i=\begin{bmatrix} 0 & 1 \\ -k_{pi} & -k_{vi} \end{bmatrix},$$

with $k_{pi}=25$, $k_{vi}=5$ for i=1 and 2 according to Eqns.(6-3-4a) and (6-3-4b).

Since P_i functions as the gain of the adaptive controller and is proportional to the size of the residual set and Q_i is associated with the rate of convergence of tracking errors and they are also restricted by the Lyapunov equation

$$A_iP_i + P_iA_i = -Q_i,$$

the determination of P_i and Q_i is a compromise between the transient response and steady state errors of the closed loop system. After a trade-off between P_i and Q_i, Q_i are set as

$$Q_1 = \begin{bmatrix} 50 & 0 \\ 0 & 8 \end{bmatrix}$$

and

$$Q_2 = \begin{bmatrix} 75 & 0 \\ 0 & 7 \end{bmatrix}.$$

Using the Lyapunov equation, P_1 and P_2 can be computed as

$$P_1 = \begin{bmatrix} 30 & 1 \\ 1 & 1 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 32.5 & 1.5 \\ 1.5 & 1 \end{bmatrix}.$$

One difference between ADA1 and ADA2 is that in Eqn.(4-2-12) x_i are the filtered error system states (see Eqn.(4-2-6a)) while in Eqn.(5-4-4) y_i is the error state itself. The final implementation of the adaptive control torque in the ADA1 is given by substituting the solution of Eqn.(4-2-17) into Eqn.(4-2-10) and then substituting Eqn.(4-2-10) into Eqn.(4-2-6d) to obtain u_{ai}.

According to Eqn.(4-2-17) and recalling $\phi_i=\bar{\theta}_i-\hat{\theta}_i$ it follows that

$$\dot{\hat{\theta}}_i = - \beta_i\hat{\theta}_i + \gamma_i\, b_i^T P_i x_i \delta_i + \beta_i\bar{\theta}_i. \qquad (6\text{-}3\text{-}10)$$

For this differential equation the contribution of the constant input term $\beta_i\bar{\theta}_i$, which is an unknown, will be a constant. In the simulations it is chosen as $\bar{\theta}_i=0$. Moreover, the filter parameters α_i in Eqn.(3-4-5a) are set as $\alpha_1=\alpha_2=0.2$ which defines a small time constant for the filter to have a rapid response. The controller parameters γ_i, which are proportional to the gains of adaptation law Eqn.(4-2-17) as well, are set as $\gamma_1=0.01$, $\gamma_2=2.5$ respectively to give a reasonable gain in Eqn.(4-2-17). Another controller parameter β_i in control law Eqn.(4-2-17) functions as a weighting factor which balances the old estimates and new estimates. In the simulation they are set as $\beta_1=\beta_2=0.2$ to give a proper update speed.

6-4-2. Design Parameter Determination for ADA2

The P_i in ADA2 are determined by the system realization of subsystem error equations Eqns.(5-4-15), (5-4-16). In the simulation the following choices in state realization are made

$$A_i = \begin{bmatrix} 0 & 1 \\ -k_{pi} & -k_{vi} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -25 & -5 \end{bmatrix}$$

for i=1 and 2, which are Hurwitz matrices (see Appendix A). Based on the same consideration mentioned in the previous section α_i is set as $\alpha_1=\alpha_2=0.2$, which, according to Eqn.(5-4-9b), gives

$$b_i = \begin{bmatrix} 1 \\ \alpha_i - k_{vi} \end{bmatrix} = \begin{bmatrix} 1 \\ -4.8 \end{bmatrix}.$$

In view of Eqn. (5-4-9c)

$$h_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

for i=1 and 2. It can be shown that the pairs $[A_i, b_i]$ and $[A_i, h_i]$ are controllable and observable. By this state realization the transfer function Eqn.(5-4-3) becomes

$$w_i(s) = h_i^T(sI-A_i)^{-1}b_i = \frac{s+0.2}{s^2+5s+25}.$$

It is easy to see that $w_i(s)$ is strictly positive real as

$$\mathrm{Re}[w_i(j\omega)]=\frac{5+4.8\omega^2}{(25-\omega^2)^2+25\omega^2}>0$$

for all real ω. The positive definite matrices U_i of the Lyapunov equations (Eqns.(5-4-15) and (5-4-16)), which are associated with the error system defined by are the triple $[A_i, b_i, h_i]$ given by Eqn.(5-4-9), are set to be

$$U_i=\begin{bmatrix} 30 & 0 \\ 0 & 20 \end{bmatrix}$$

under the same consideration stated in Section 6-4-1. The small number κ_i defined in Eqn.(5-4-15) is determined as κ_i=0.001. In accordance with the Kalman-Yacubovitch lemma, for A_i, b_i, h_i, U_i, and κ_i given above, P_i and v_i satisfying the Lyapunov equation

$$A_i^TP_i+P_iA_i=-v_iv_i^T - \kappa_iU_i$$

$$P_ib_i=h_i$$

can be obtained as

$$P_i=\begin{bmatrix} 2.08 & 0.22 \\ 0.22 & 0.0468 \end{bmatrix},$$

and

$$v_i=\begin{bmatrix} 3.35 \\ 0.064 \end{bmatrix}.$$

In the simulations, for simplification, the design parameters Γ_i in Eqn.(5-4-18) are chosen as positive scalars γ_i instead of matrices. Based on the same reasons mentioned in Section 6-4-1, γ_i and β_{oi} are chosen as the same as those in Section 6-4-1, that is, $\Gamma_1=\gamma_1=0.05$, $\Gamma_2=\gamma_2=12.5$ and $\beta_{o1}=\beta_{o2}=0.2$.The parameters $\hat{\theta}_{oi}$ are set as small constants $\hat{\theta}_{o1}=\hat{\theta}_{o2}=0.01$ to ensure the linearity of the adaptive controllers .

6-5. SIMULATION RESULTS

During the simulations, two types of reference trajectories were considered to illustrate the performance of the proposed control approaches. By means of Trajectories Type 1 (RTJ1), which are sine wave functions with quite high frequencies, the abilities of the proposed schemes to follow fast moving trajectories is investigated. In addition the stabilized functions and changing patterns of estimated parameters are discussed. The second type of trajectories (RTJ2) are designed to simulate a real pick-and-load task. Simultaneously with RTJ2, a changeable payload is used to examine the robustness for the proposed adaptive control schemes.

In the model used in simulation, the true values of the robot arm's dynamic parameters are set as

$$L_1 = 0.5\text{m},$$

$$L_2 = 0.3\text{m},$$

$$m_1 = 6\text{kg}$$

$$m_2 = 4\text{kg}.$$

However, the choice of the mass of the payload will depend on the simulation purposes and will be mentioned shortly.

6-5-1. Reference Trajectories Type 1 – RTJ1

The first type of reference trajectories RTJ1 used are

$$q_{d1} = 1+\sin 3t+\sin 2t$$

$$q_{d2} = -0.5+0.8(\cos 3t+\cos 2t)$$

which are shown in Fig.6-2. In this case the mass of the robot payload was fixed as $m_L \equiv 0$kg. The a priori estimates of the system parameters are set as

$$\hat{L}_1=0.48\text{m},$$

$$\hat{L}_2=0.28\text{m},$$

$$\hat{m}_1=5.5\text{kg},$$

$$\hat{m}_2=3.5\text{kg}$$

$$\hat{m}_L=1.0\text{kg}$$

and initial values of the estimated unknown parameters $\hat{\theta}_{ij}(0)=0$ for i=1,2 and j=1,2,3 and 4.

Based on this set of estimates, using RTJ1, the position tracking errors e_1 and e_2 of the computed torque scheme are plotted in Fig.6-3 which gives significant tracking errors especially for the second joint. It shows that the maximum position error is about 0.15 rad for joint 1, and 0.6 rad for joint 2. However, by means of the adaptive control algorithm 2, using the same initial conditions and the same estimated dynamic parameters, the control results are plotted in Fig.6-4 which decreases the tracking errors caused by the computed torque approach significantly (note the different vertical scales used in the two plots). After about 8 seconds settling down, the maximum errors are 0.03 rad for joint 1 and 0.025 rad for joint 2 respectively.

In Fig.6-5 to 6-8, the estimated parameters in joints 1 and 2 for the adaptive controller are plotted. Because of the existence of the interconnections, the estimated parameters do not converge to their true values; however, they all remain bounded. Due to the influences of the interconnections among the different subsystems, the adaptive controller works in such a way that the parameters move to whatever levels are necessary to ensure the stability of each subsystem. Considering the interconnections among the subsystems as perturbations, the estimated parameters, which can also be regarded as the gains of the adaptive feedback loop, vary above certain levels with the same frequencies as that of the reference trajectories to compensate the time varying perturbation and stabilize the closed loop subsystem. Compared with the true values of these parameters, the estimated parameters are much smaller than their true values. This can be explained by the "small gain" features [18][53] of the adaptive controller when there are some interconnections among different subsystems.

6-5-2. Reference Trajectories Type 2 – RTJ2

In order to simulate real applications of robots such as undertaking pick-and-load tasks, another type of reference trajectories RTJ2, which are shown in Fig.6-9, were also used in the simulation. Specified by these types of trajectories, the robot arm is supposed to start from position 0 (q_1=0 rad, q_2=0 rad) and move through four points, which are position 1 (q_1=2.51 rad, q_2=-1.256 rad); position 2 (q_1=-1.49 rad, q_2=0.744 rad); position 3 (q_1=1.01 rad, q_2=0.244 rad) and position 4 (q_1=-1 rad, q_2=-0.755 rad) following RTJ2. To investigate the algorithms' robust features, the influence of changing payload is also investigated. The payload function is a square wave with different magnitudes as plotted in Fig.6-10. These reference trajectories and payload functions define the following task: the robot's end effector grasps an object weight 2 kg at position 0 and takes it to position 1 following the given track. After holding for 0.36 second and releasing the object at that point, it moves to position 2 where it picks up the second object weighing 1 kg. Carrying this object to position 3, the arm unloads it at that point and finally goes to position 4 without any load. In this motion each movement takes the same time (about 3.14 seconds) but different distances are travelled; the maximum velocity and acceleration occur in travelling from position 1 to position 2.

Using reference trajectories RTJ2, several groups of a priori estimates of the unknown dynamic parameters are used in the simulation to see the performance of proposed adaptive controllers under different initial estimates. Utilizing the same parameters the simulations using computed torque scheme were also presented to compare with the proposed methods. Five groups of the a priori estimates are shown in Table. 6-1.

			A priori estimates of parameters				
Parameter true values			Group 1	Group 2	Group 3	Group 4	Group5
L_1 (m)	0.5	$\hat{L}_1$	0.55	0.48	0.50	0.47	0.53
L_2 (m)	0.3	$\hat{L}_2$	0.32	0.28	0.30	0.32	0.33
m_1 (kg)	6.0	$\hat{m}_1$	6.5	5.5	6.0	5.2	4.0
m_2 (kg))	4.0	$\hat{m}_2$	4.3	4.5	4.0	4.5	5.0
m_L (kg)	shown by Fig.6-10	$\hat{m}_L$	0.0	1.0	1.0	0.8	0.0

Table.6-1. Estimated robot parameter groups used in the simulations

The first group of parameters presents a parameter "over-estimated" situation, i.e., all initial estimates for the robot body are larger than their true values except that the estimate of the payload is $\hat{m}_L$=0.0 kg. Using this set of a priori estimates the position tracking errors of the computed torque scheme are plotted in Fig.6-11 which shows the maximum position errors to be about 0.3 rad for joint 2. The largest errors occur at the moments t =1.7s and t =3.1s when the reference trajectory 2 moves with its maximum acceleration and deceleration. These errors are caused mainly by the error in the estimate of the heavy payload (see Fig.6-10 which shows that in this interval it is 2 kg). Using the same set of initial estimates, the position tracking errors of ADA1 and ADA2 schemes are shown in Fig.6-12 and Fig.6-13 respectively. These plots show that the tracking error of the second joint in the computed torque scheme has been greatly reduced to 0.025 rad maximum. For the first joint the maximum error is almost the same but the error variance has been reduced significantly especially during the motion from point 0 to point 1. It also can be seen that ADA2 scheme shows a smoother result than ADA1 especially for the first joint.

Using the second group of a priori estimates ($\hat{L}_1$=0.48m, $\hat{L}_2$=0.28m, $\hat{m}_1$=5.5kg, $\hat{m}_2$ =3.5kg and $\hat{m}_L$=1.0kg) which is an "under estimated" case for the robot dynamic parameters, the performances of the computed torque, ADA1 and ADA2 schemes are shown by Fig.6-14, 6-15 and 6-16 respectively. In this group of plots, adaptive controllers give a much improved performance over the computed torque scheme. The tracking errors under adaptive control in both joints are much smaller than those of the computed torque control.

The next simulation is for the ideal case in which the initial estimates exactly match the robot dynamic parameters. The initial estimates of the payload are set as $\hat{m}_L$=1.0 kg for all schemes, i.e., the computed torque and the two adaptive schemes. From Fig.6-17 it can be seen that during the time interval $t \in$ [6.8s, 10.5s], the computed torque scheme gives perfect performance with zero tracking errors for both joints. This is expected and is caused by perfect matches of both robot body parameters and payload parameters (in this interval $\hat{m}_L = m_L$). (This also shows the model used in the simulation and algorithm software is accurate). However, the tracking errors beyond this interval are much worse due to the error in the estimate of payload, which means that the computed torque scheme is very sensitive to the payload error. At the same time, the tracking errors of controllers ADA1 and ADA2 are shown in Fig.6-18 and 6-19. It can be seen that during the time interval $t \in$ [6.8s, 10.5s], they are definitely not as good as the computed torque scheme's performance, but give smaller errors than the computed torque scheme over the remaining movements.

This simulation illustrates that with fixed gains the performance of the computed torque law strongly depends on the robot system parameters. Even in the case where the robot dynamic parameters are well known, time-varying payloads may influence performance significantly and a controller set for a certain payload condition may give quite large tracking errors whenever the payload is changed. The adaptive control schemes did not show as small tracking errors during the time interval when the parameters and the estimates were perfectly matched (for reasons which include the ignoring of interactions between subsystems and the fact that the estimator is not able to track the step change in payload rapidly, etc.), but they do give consistent acceptable performances. This also means that these adaptive controllers are robust to uncertainties caused by changing payloads and system parameters.

Two more simulations were undertaken which present the "over-under estimated" cases for initial estimations of dynamic parameters. The fourth group initial estimation, is given by

$$\hat{m}_1=5.2\text{kg}<m_1=6.0\text{kg},$$

$$\hat{m}_2=4.5\text{kg}>m_2=4.0\text{kg},$$

$$\hat{L}_1=0.47\text{m}<L_1=0.5\text{m},$$

$$\hat{L}_2=0.32\text{m} >L_2=0.3\text{m},$$

and

$$\hat{m}_L=0.8 \text{ kg}.$$

It can be seen that the mass of the first link and the length of the second link are both over-estimated and mass of the second link and the length of the first link are both under-estimated. With this set of initial estimates, the tracking errors of the computed torque and ADA2 are plotted in Fig.6-20 and 6-21. In the final case, the estimates were set as

$$\hat{m}_1=4.0\text{kg}<m_1=6.0\text{kg},$$

$$\hat{m}_2=5.0\text{kg}>m_2=4.0\text{kg},$$

$$\hat{L}_1=0.53\text{m}>L_1=0.5\text{m},$$

$$\hat{L}_2=0.33\text{m}>L_2=0.3\text{m},$$

and

$$\hat{m}_L=0.0\ \text{kg}.$$

The results are shown in Fig.6-22 and 6-23.

From these plots, it is found that in both cases the maximum tracking errors for the computed torque scheme are 0.04 rad and 0.25 rad for joints 1 and 2 respectively. However, the adaptive control scheme 2 gives maximum tracking errors of 0.03 rad for both cases and both joints.

These simulation results show that whenever the robot dynamic parameters are unknown, then under the same initial conditions the adaptive control algorithms proposed in Chapters 4 and 5 give consistently improved performances compared with the computed torque method, especially in cases when the payload is changeable. This is achieved by the self-adjusting capability of the adaptive controller. As the controller is designed based on the Lyapunov direct method, as soon as the tracking error and the parameter error increase the controller will adjust its parameters so that the Lyapunov function will decline to maintain the stability of the closed loop system and decrease the tracking errors. Since the computed torque controller has fixed gains and parameters, it only works as well in cases where its parameters closely match the real parameters of the controlled robot.

6-6. SUMMARY

In this chapter, using the equations of motion of a SCARA manipulator as an example, the design procedures of the adaptive controllers proposed in this thesis were reviewed. Firstly the dynamic model of the system under study was introduced. Detailed design processes were presented which included the system structure and the determination of parameters for the adaptive controllers.

Simulation results using the proposed adaptive control schemes were also presented. In the simulations the effects of two types of reference trajectories and time varying payloads were examined. Under the assumptions that the real dynamic parameters are unknown, a priori estimates are used in the simulations. With different initial estimates of these parameters the simulations were carried out under different conditions. Compared with the results of the computed torque scheme, the proposed adaptive control algorithms give substantially better performance and exhibit considerable robustness under a wide variety of conditions.

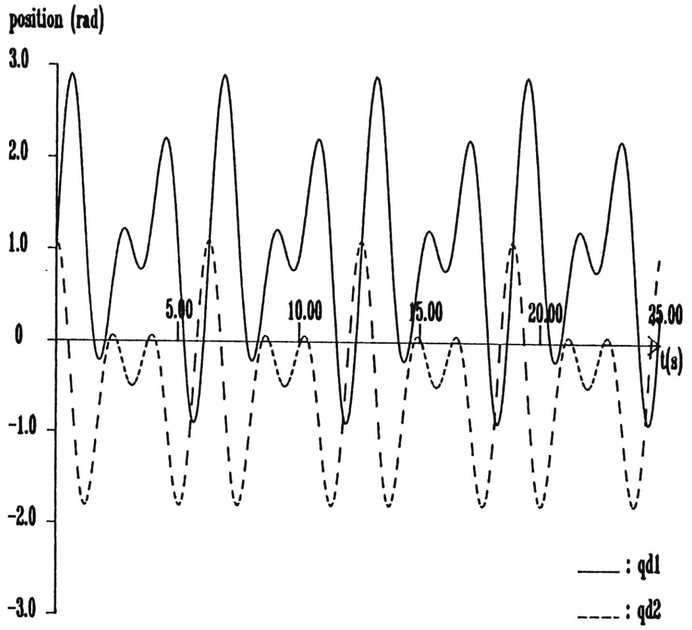

Fig.6-2. Reference Trajectories Type 1 (RTJ1) used in the simulations

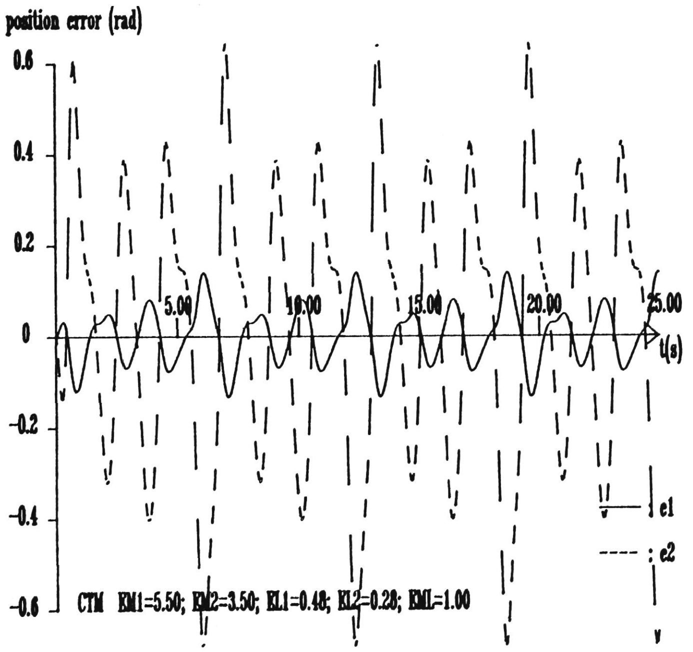

Fig.6-3. Position tracking errors of the Computed Torque Scheme in following RTJ1

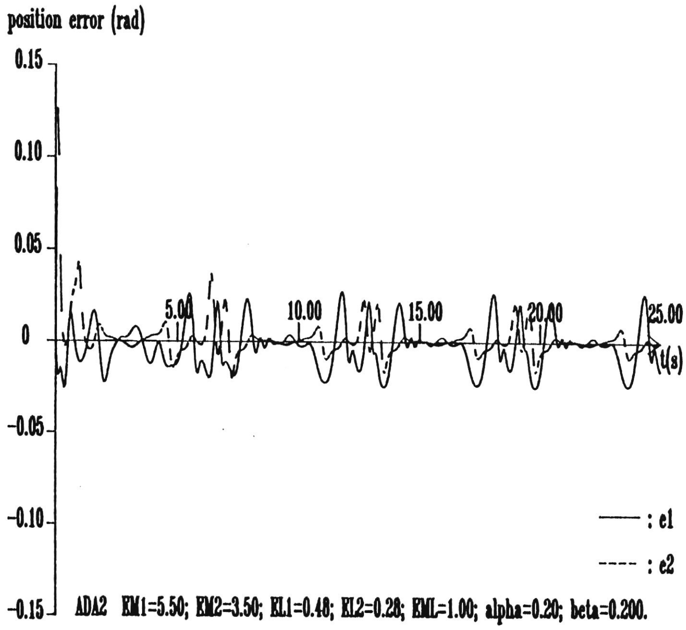

Fig.6-4. Position tracking errors of the ADA2 Scheme in following RTJ1

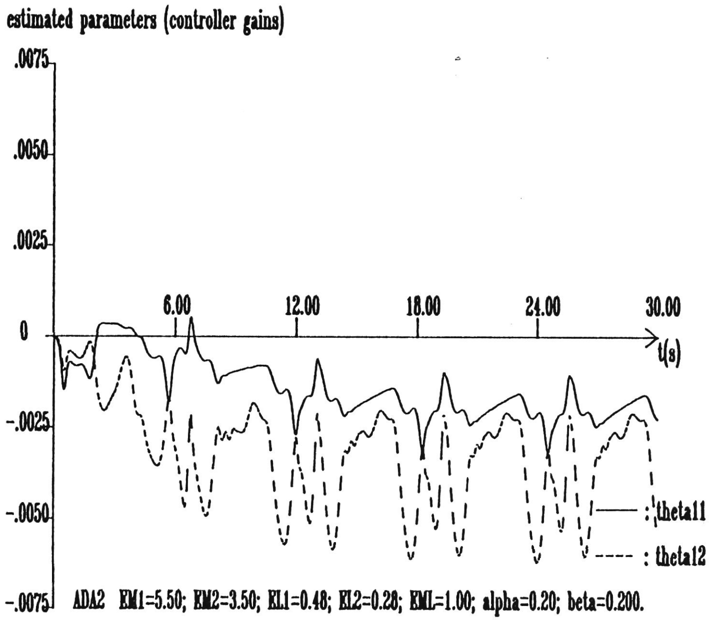

Fig.6-5. Parameter estimates $\hat{\theta}_{11}$ (theta 11) and $\hat{\theta}_{12}$ (theta 12) in subsystem 1 of ADA2 scheme when RTJ1 used

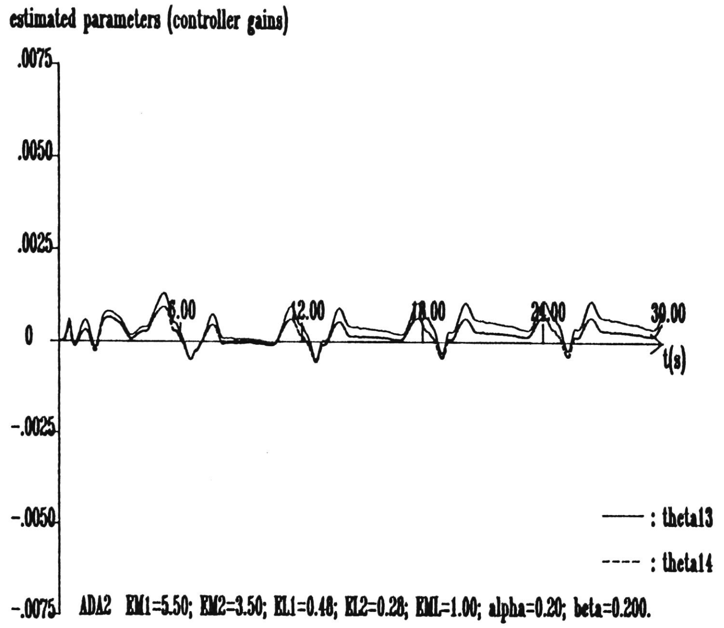

Fig.6-6. Parameter estimates $\hat{\theta}_{13}$ (theta 13) and $\hat{\theta}_{14}$ (theta 14) in subsystem 1 of ADA2 scheme when RTJ1 used

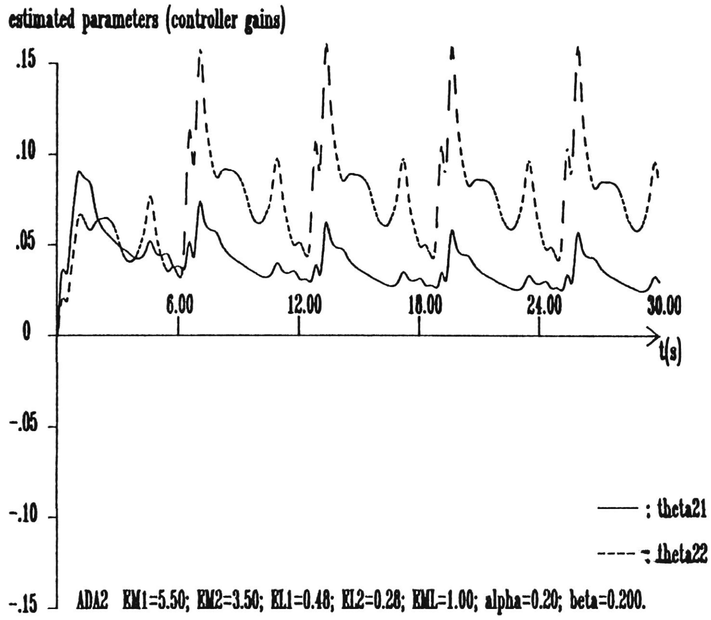

Fig.6-7. Estimated parameters $\hat{\theta}_{21}$ (theta 21) and $\hat{\theta}_{22}$ (theta 22) in subsystem 2 of ADA2 scheme when RTJ1 used

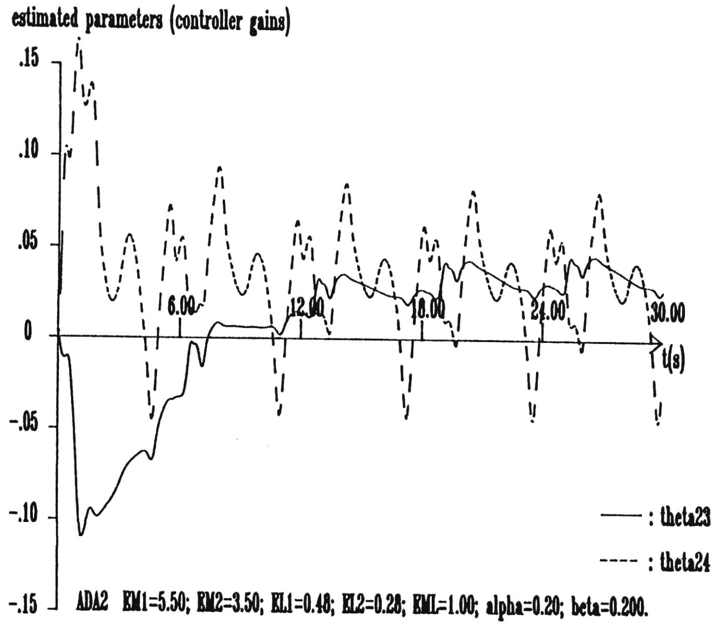

Fig.6-8. Parameter estimates $\hat{\theta}_{23}$ (theta 23) and $\hat{\theta}_{24}$ (theta 24) in subsystem 2 of ADA2 scheme when RTJ1 used

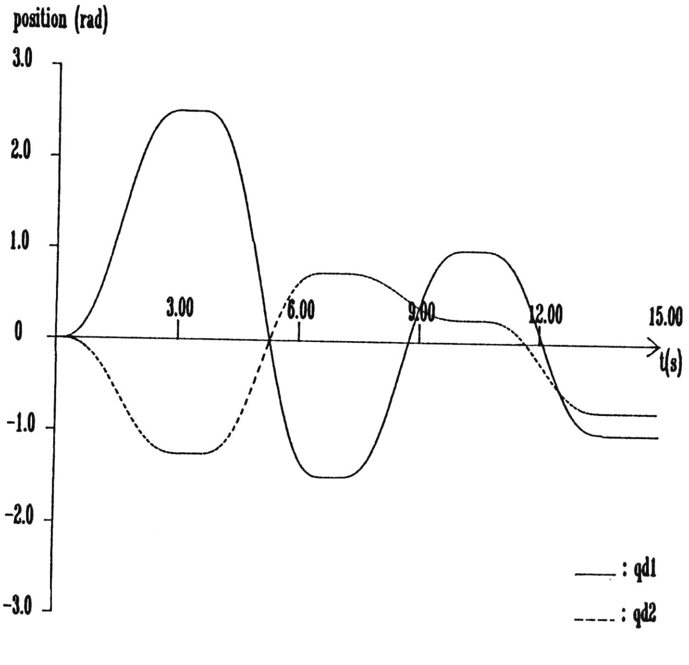

Fig.6-9. Reference Trajectories Type 2 (RTJ2) used in the simulations

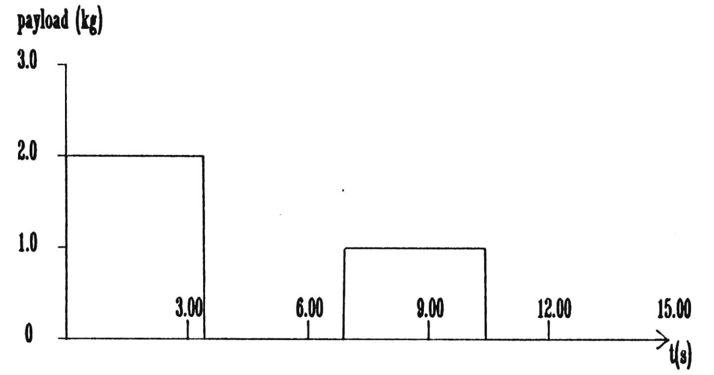

Fig.6-10. Time-varying load function used in the simulation which is applied simultaneously with reference trajectories RTJ2

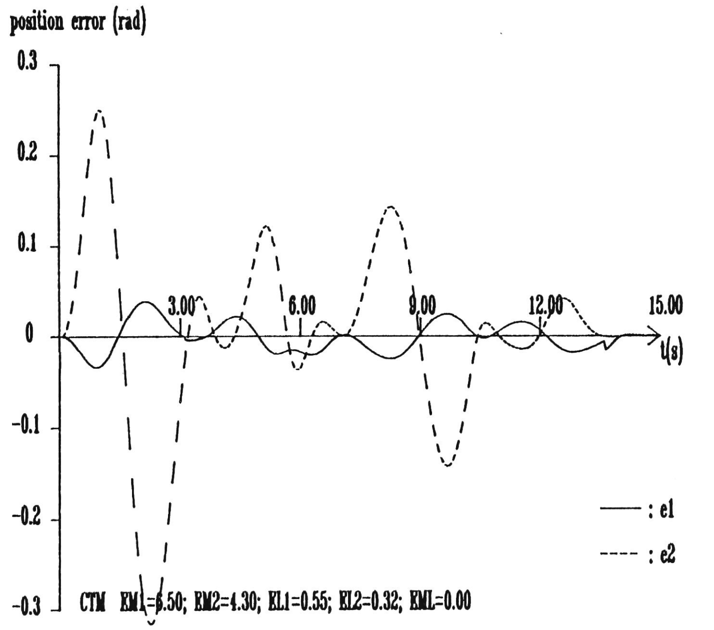

Fig.6-11. Position tracking errors of the Computed Torque Scheme using the 1st group of initial estimates in following RTJ2

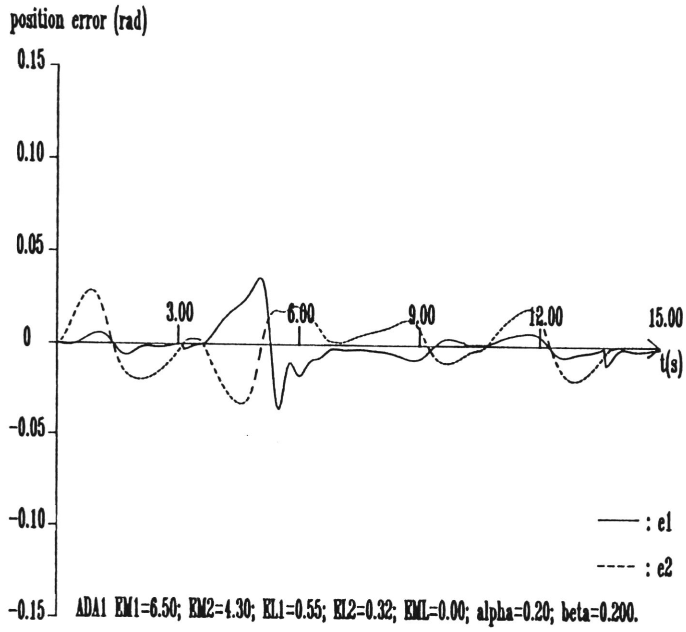

Fig.6-12. Position tracking errors of Scheme ADA1 using the 1st group of initial estimates in following RTJ2

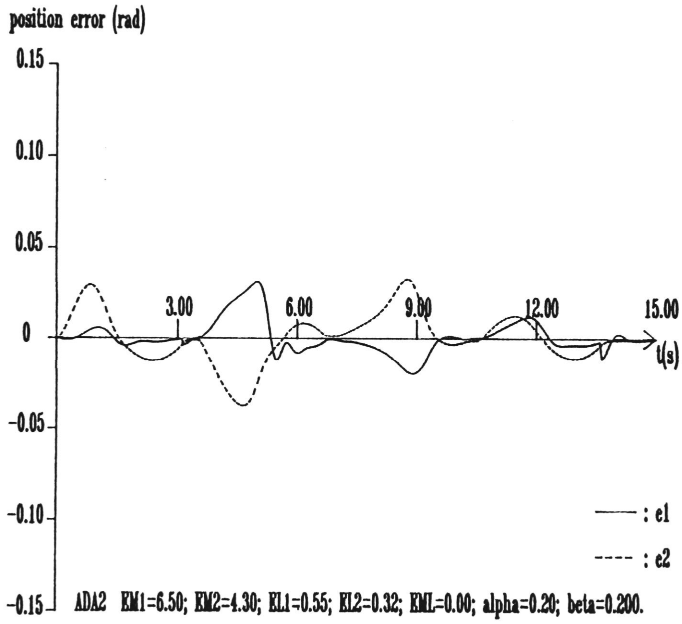

Fig.6-13. Position tracking errors of Scheme ADA2 using the 1st group of initial estimates in following RTJ2

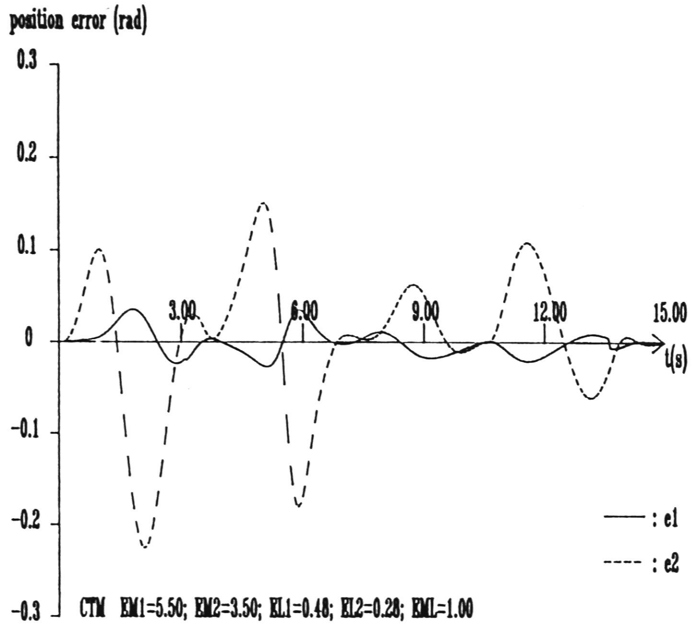

Fig.6-14. Position tracking errors of the Computed Torque Scheme using the 2nd group of initial estimates in following RTJ2

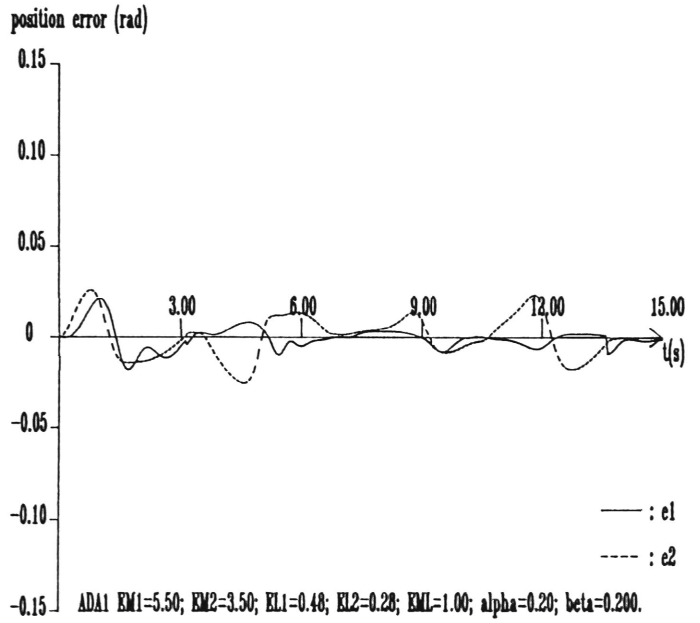

Fig.6-15. Position tracking errors of Scheme ADA1 using the 2nd group of initial estimates in following RTJ2

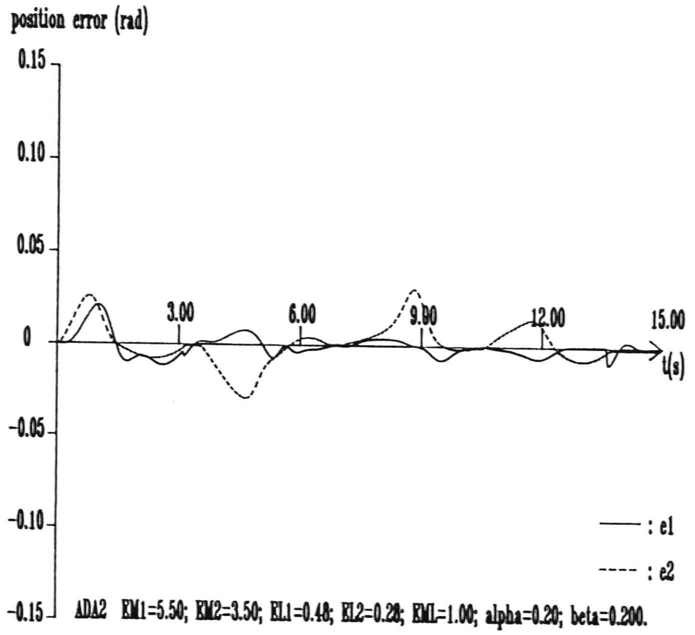

Fig.6-16. Position tracking errors of Scheme ADA2 using the 2nd group of initial estimates in following RTJ2

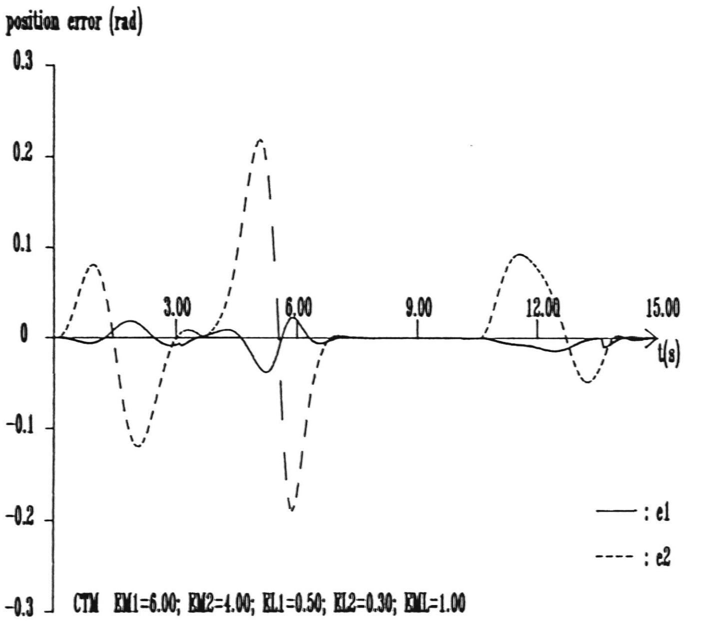

Fig.6-17. Position tracking errors of the Computed Torque Scheme using the 3rd group of initial estimates in following RTJ2

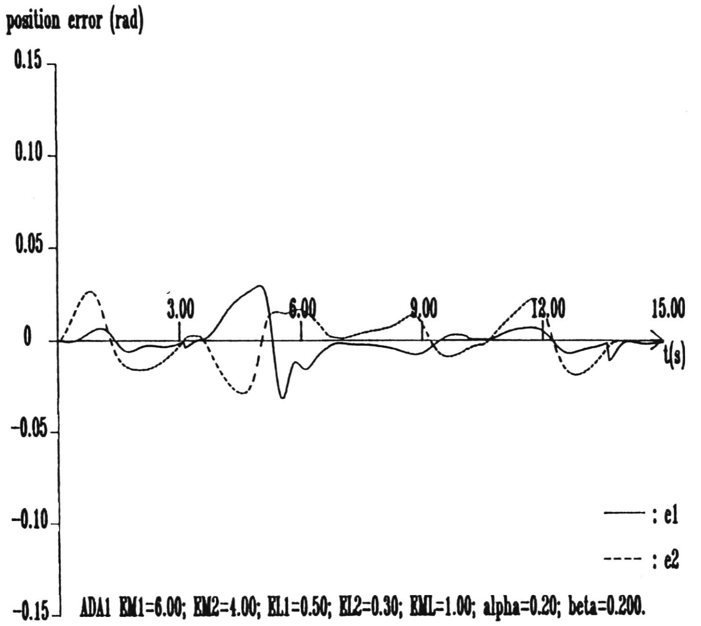

Fig.6-18. Position tracking errors of Scheme ADA1 using the 3rd group of initial estimates in following RTJ2

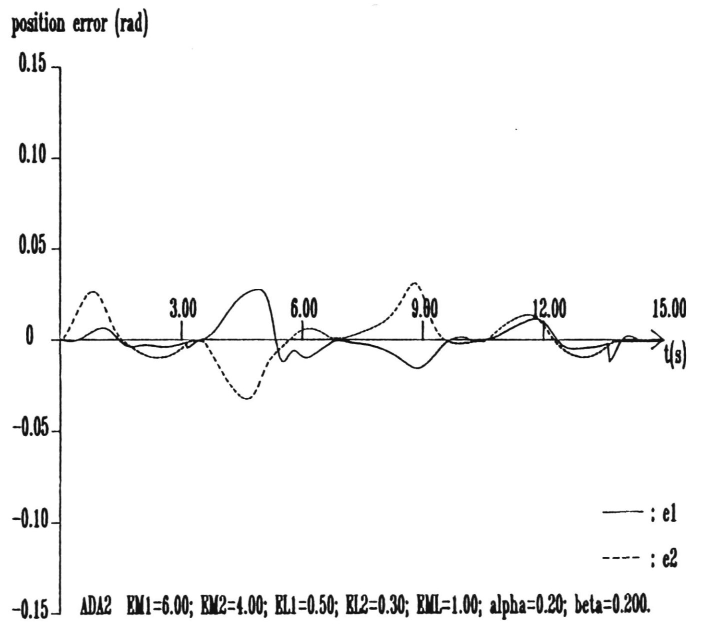

Fig.6-19. Position tracking errors of Scheme ADA2 using the 3rd group of initial estimates in following RTJ2

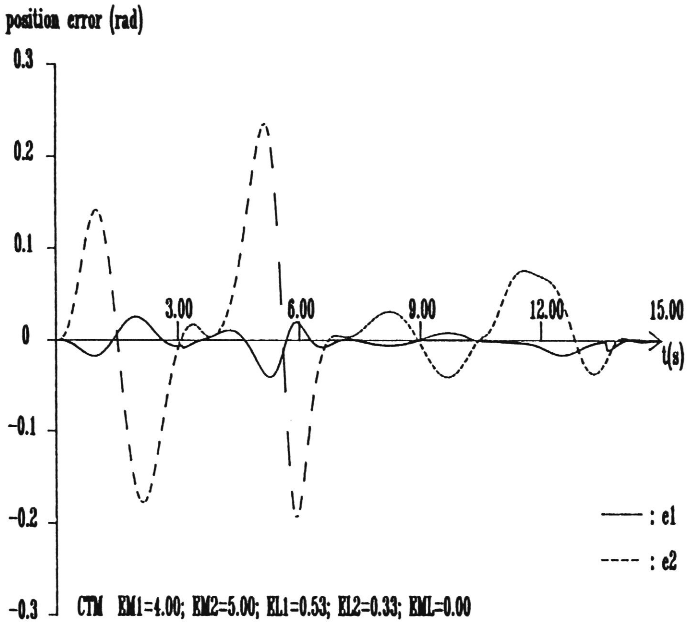

Fig.6-20. Position tracking errors of the Computed Torque Scheme using the 4th group of initial estimates in following RTJ2

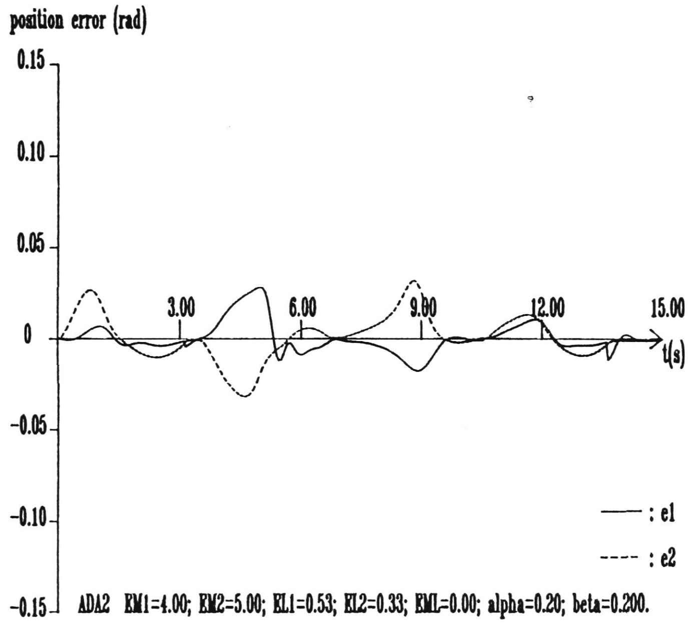

Fig.6-21. Position tracking errors of Scheme ADA2 using the 4th group of initial estimates in following RTJ2

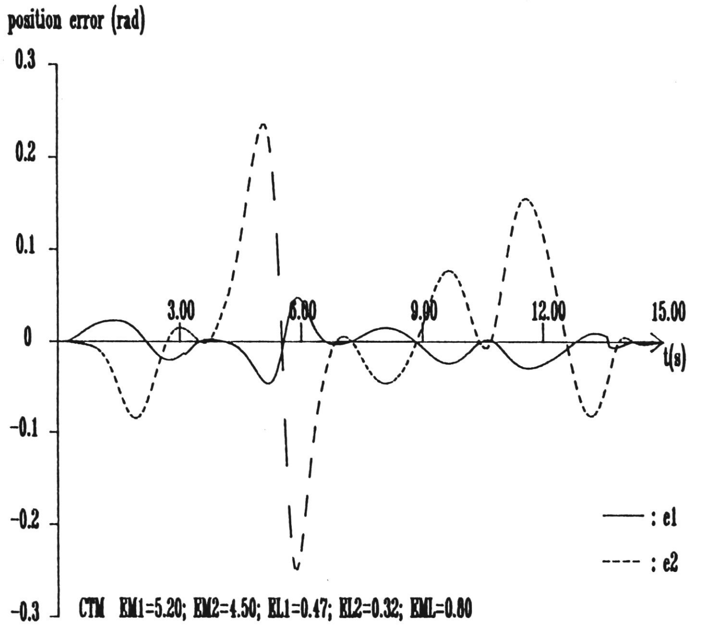

Fig.6-22. Position tracking errors of the Computed Torque Scheme using the 5th group of initial estimates in following RTJ2

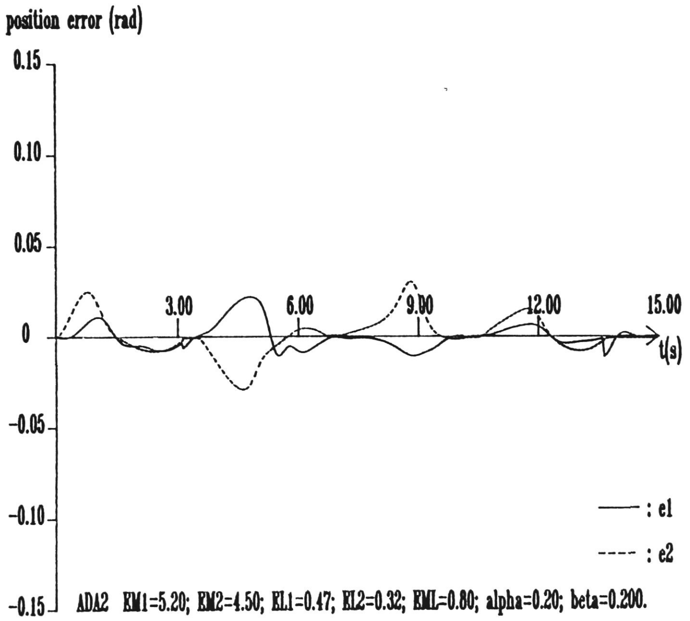

Fig.6-23. Position tracking errors of Scheme ADA2 using the 5th group of initial estimates in following RTJ2

Chapter 7.

CONCLUSIONS

7–1. CONCLUSIONS

This thesis proposes two novel adaptive control algorithms for the trajectory tracking control of robotic manipulators for which the dynamic parameters are unknown or partially unknown. The controllers are based on a two component structure: a computed torque control law plus an adaptive control law. The computed torque control law is implemented using a set of a priori estimates of the unknown dynamic parameters. For the error system dynamics obtained, the adaptive control scheme is designed utilising a novel decentralized error system architecture in adaptive robot controller design. Unlike the normal decentralized system in which the controller of each subsystem uses only local feedback , this system structure utilises partial global feedback to improve the tracking performance. This system configuration allows other global tracking information which has not been taken into account to be treated as interconnections between the different subsystems.

Within this system architecture, the tool for the adaptive controller design is the Lyapunov direct method. The resultant controllers are shown to be robust despite the interconnections from other subsystems and the existence of bounded uncertainties. The robustness is in the sense that the tracking errors are bounded inside a residual set which is proportional to the unknown boundedness of the interconnections and the uncertainties.

The proposed schemes also offer other advantages as follows:

1). Unlike some adaptive schemes which require the inverse of the estimated inertial matrix $\hat{D}^{-1}(q)$ to be computed on-line using the updated parameter estimates and the measured positions of the robot joints, the schemes proposed here only need the diagonal elements to be calculated.

2). In some adaptive schemes, the parameter estimate algorithm must make sure that the obtained estimate of the inertial matrix $\hat{D}(q)$ is a non-singular matrix so that its inverse always exists. This restriction is quite strong in the sense that it requires either that the bounds of the unknown parameters are known by the estimator so that as soon as the estimates arrive at these bounds the estimator should stop updating, or that the rank of $\hat{D}(q)$ should be checked from time to time which is quite difficult especially for robots with several degrees of freedom. This restriction does not exist at all in the schemes presented here since: i) the estimates used to form $\hat{D}(q)$ are set by a group of values which are constants during the real-time control and; ii) as stated in Chapter 3, it is always possible to find a group of estimates so that the existence of $\hat{D}^{-1}(q)$ can be guaranteed. Basically, this is achieved by the proposed two-component controller architecture in which the estimate $\hat{D}(q)$ is only invoved in the non-adaptive component and adaptation is accomplished by another adaptive control component.

3). Due to the parameter estimation errors, the resultant error equations of the computed torque scheme are a group of second order linear equations forced by nonlinear functions of q, $\dot{q}$ and $\ddot{q}$. Direct adaptive controller design utilising these equations will need the measurements of q, $\dot{q}$ and $\ddot{q}$. In practice, measuring the acceleration vector $\ddot{q}$ may cause technical difficulties or need acceleration sensors. In the proposed schemes, this problem was overcome by feeding the velocity signals into a linear operator. The linear operator used in the second algorithm has an additional function as shown in Chapter 5. This operator is used to avoid the explicit measurement of the accelerations as well as to introduce an additional zero into the error systems to satisfy the strictly positive real condition.

4). In the stability and convergence analysis, quantitative results on the boundedness of the tracking errors and parameter estimate errors are obtained, as well as their convergence rates. It has been shown that the size of the bounded residual set to which the errors finally converge are proportional to the magnitudes of the interconnections and the maximum boundedness of the trajectories applied. This result reveals quantitative relationships between the ignored interconnections and the resultant position and velocity tracking errors. Graphical illustrations of the boundedness and convergence rates are also presented. The analysis also shows procedures for determining controller design parameters.

The two adaptive control algorithms, presented in Chapters 4 and 5 respectively, are similar except that in method 2 a dead zone in the parameter update law, given by Eqn.(5-4-18), is introduced. The principle difference between the two methods results from the distinctive way of introducing the additional zero into the error system to satisfy the strictly positive real condition. In the first algorithm (see Theorem 5-1) in addition to the necessary assumptions the conditions (C-3)–(C-6) are required to ensure the bounded interconnections in the stability and convergence analysis. Physically, the satisfaction of these conditions means that the computed torque law, as one of the two control components, should be implemented based on a set of parameter estimates which are quite close to the true values of the unknown parameters. Based on this control component a further improvement of tracking performance can be achieved by the adaptive control law. However, in the investigation of the second algorithm, it has been shown that the boundedness of the interconnections depends on the boundedness of the overall system states (positions and velocities of each joints). Based on this fact, the second algorithm developed is based on a more complete theoretical analysis. In this algorithm the conditions (C-3)–(C-6) were removed. This facilitates its practical application and is an achievement based on analysis of the first algorithm and properties of the interconnections. The second algorithm introduces a dead zone and requires that M is positive definite. The simulations show that these are in practice not restrictive conditions.

Also, compared with the first algorithm, the second has the advantage that in the error system equation the system states are the real position and velocity errors instead of the filtered errors. This makes it possible to prove the boundedness of both position and velocity tracking errors in Theorem 5-2.

To justify the theoretical results, several simulation results using the proposed control algorithms are presented in the thesis. It has been shown, in Chapter 6, that the control schemes give excellent tracking performance compared with the pure computed torque scheme. In addition, improved control results, i.e., smaller tracking errors plus robustness despite parameter errors and time varying payload, are demonstrated.

6–2. RECOMMENDATIONS FOR FUTURE WORK

The equations of motion employed in this thesis most closely represent the dynamics of direct drive robot arms in which all friction torques are ignored. Although uncertainties have been taken into account, it is possible to consider the dynamics of actuators such as DC servo motors and the friction torques such as dynamic friction, viscous friction and Coulomb friction and set up equations which are closer to the dynamic behaviour of controlled robots. Further useful work could use the ideas presented in this thesis to investigate the trajectory following control problems of robots with more detailed dynamics information.

For most industrial commercial robot arms, joint independent controllers using local feedback are used. This control system structure represents a decentralized system configuration. Especially for non-direct drive robots in which the transmissions are mounted in each joint, the high gear ratios will make couplings between the joints much weaker. For this kind of robot arm, the analysis method proposed in this thesis may be used to reveal the quantitative relationship between the tracking error boundedness and some effects related to mechanical structures such as gear ratios etc.. Further work could be performed to extend the methods proposed in this thesis to design a pure local feedback using a decentralized system adaptive controller for non-direct drive robots.

In robot dynamic control, some relatively new research areas such as force control and compliance control have been exploited to meet strong demands from practical industrial applications. Further investigations may succeed in extending the control methods proposed in this thesis to develop decentralized system adaptive force control or compliance control strategies.

APPENDIXES

APPENDIX A

Definition 1: Positive Real Functions.[30]

A rational function w(s) of the complex variable $s=a+j\omega$ is positive real if

i. w(s) is real for all real s;

ii. $Re[w(s)] \geq 0$ for all $Re[s]>0$.

Definition 2: Strictly Positive Real Function. [30]

A rational function w(s) of complex variable $s=\alpha+j\omega$ is strictly positive real if

i. w(s) is real for all s;

ii. w(s) has no poles in the closed right half plane $Re[s] \geq 0$;

iii. $Re[w(j\omega)]>0$ for all ω.

Definition 3: Hurwitz Polynomial.

An n-order polynomial h(s) with real coefficients is defined as the Hurwitz polynomial if all its eigenvalues λ_i (i=1, 2, ..., n) have negative real parts, i.e.,

$$Re[\lambda_i]<0, \quad (i=1, 2, ..., n) \quad \text{for} \quad \forall\ h(\lambda_i)=0.$$

Definition 4: Hurwitz Matrix.

A real nxn matrix M is a Hurwitz matrix if its all eigenvalues have negative real parts, i.e.,

$$Re[\lambda_i(M)]<0, \quad (i=1, 2, ... n) \quad \text{for} \quad \forall\ \lambda_i I-M=0$$

APPENCIX B

The Kalman Yacubovitch Lemma (Positive Real Lemma). [33][64]

Given a stable matrix A, a symmetric matrix U>0, vectors $b \neq 0$, and h, and scalars $\gamma \geq 0$ and $\kappa > 0$ such that the pair (A, b) is controllable, then a sufficient and necessary condition for the existence of a solution matrix P>0 and a vector v of the equation

$$A^T P + PA = -vv^T - \kappa U$$

$$Pb - h = \sqrt{\gamma} v$$

is that κ is small enough and scalar function

$$w(s) = \gamma + h^T (sI - A)^{-1} b$$

is strictly positive real, i.e., $\mathrm{Re}[w(j\omega)] > 0$ For all ω.

APPENDIX C

Proof of Theorem 5-1:

For error state equation Eqns.(5-4-7) and (5-4-8), in which i=1, 2, ..., n, a candidate for the Lyapunov function [27] is chosen as:

$$v(y, \phi) = \sum_{i=1}^{n} \pi_i \left(\frac{1}{2} y_i^T P_i y_i + \frac{1}{2} \phi_i^T \Gamma_i^{-1} \phi_i\right). \tag{AC-1}$$

Its total derivative along the solution trajectory of Eqn.(5-4-7) is

$$\dot{v}(y,\phi) = \sum_{i=1}^{n} \pi_i \left[-\frac{1}{2} y_i^T (A_i^T P_i + P_i A_i) y_i + y_i^T P_i b_i \phi_i^T \delta_i + \phi_i^T \Gamma_i^{-1} \dot{\phi}_i - y_i^T P_i b_i \eta_i \right]. \tag{AC-2}$$

In view of Eqn.(5-4-15) and (5-4-16), (AC-2) becomes

$$\dot{v}(y,\phi) = \sum_{i=1}^{n} \pi_i \left(-\frac{1}{2} y_i^T v_i v_i^T y_i - \frac{1}{2} \kappa_i y_i^T U_i y_i - \beta_i \phi_i^T \phi_i - \beta_i \phi_i^T \theta_i - y_i^T P_i b_i \eta_i\right)$$

$$\leq \sum_{i=1}^{n} \pi_i \left(-\frac{1}{2} \kappa_i \min\lambda(U_i) \|y_i\|^2 - \beta_i \|\phi_i\|^2 - \beta_i \phi_i^T \theta_i + |\eta_i| \, \|P_i b_i\| \, \|y_i\|\right). \tag{AC-3}$$

Consider Eqn.(5-4-19), i. e.,

$$\sigma_i = \frac{1}{2} \kappa_i \min\lambda(U_i),$$

then Eqn.(AC-3) becomes

$$\dot{v}(y,\phi) \leq \sum_{i=1}^{n} \pi_i \left(-\sigma_i \|y_i\|^2 - \beta_i \|\phi_i\|^2 - \beta_i \phi_i^T \theta_i + |\eta_i| \, \|P_i b_i\| \, \|y_i\|\right). \tag{AC-4}$$

As $e_j=q_{dj}-q_j$, and from Eqns.(5-4-4), (5-4-5) and (5-4-6), it follows that $y_{qj}=y_{dj}-y_j$, and $\|y_{qj}\| \le \|y_{dj}\|+\|y_j\|$ so that Eqn.(5-4-10) can be rewritten as

$$|\eta_i| \le \sum_{j=1}^{n} \chi_{ij} \|y_{qj}\|+c_{ij} \le \sum_{j=1}^{n} \chi_{ij} \|y_j\|+\chi_{ij} \|y_{dj}\|+c_{ij}.$$

Taking it into account, Eqn.(AC-4) becomes

$$\dot{v}(y,\phi) \le \sum_{i=1}^{n} \pi_i \,[-\sigma_i\|y_i\|^2+\|P_ib_i\| \, \|y_i\| \sum_{j=1}^{n} \chi_{ij} \|y_j\|-\beta_i\|\phi_i\|^2-\beta_i\phi_i{}^T\theta_i$$

$$+\|P_ib_i\| \, \|y_i\| \sum_{j=1}^{n} (\chi_{ij} \|y_{dj}\|+c_{ij})]$$

$$\le \sum_{i=1}^{n} \pi_i \,[-\sigma_i\|y_i\|^2+a_{ii}\|P_ib_i\| \, \|y_i\|^2+ \sum_{j=1 j\neq i}^{n} \chi_{ij} \|P_ib_i\| \, \|y_i\| \, \|y_j\| - \beta_i\|\phi_i\|^2$$

$$- \beta_i\phi_i{}^T\theta_i+ \sum_{j=1}^{n} (\chi_{ij} \|y_{dj}\|+c_{ij}) \|P_ib_i\| \, \|y_i\|].$$

Consider Eqn.(5-4-20), which is $r_{ij} = (\chi_{ij}+\frac{c_{ij}}{\|y_{dj}\|}) \|P_ib_i\|$, then the inequality above can be rewritten as:

$$\dot{v}(y,\phi) \le -\sum_{i=1}^{n} \pi_i \left(\frac{\sigma_i}{2}-r_{ii}\right) \|y_i\|^2+ \sum_{i=1}^{n} \sum_{j=1 j\neq i}^{n} \pi_i r_{ij} \|y_i\| \, \|y_j\|$$

$$+ \sum_{i=1}^{n} \pi_i \left(- \frac{\sigma_i}{2} \|y_i\|^2- \beta_i\|\phi_i\|^2- \beta_i\phi_i{}^T\theta_i+ \sum_{j=1}^{n} r_{ij} \|y_{dj}\| \, \|y_i\|\right). \qquad \text{(AC-5)}$$

As the second term in the right side of Eqn.(AC-5) can be denoted by:

$$\sum_{i=1}^{n} \sum_{j=1 j\neq i}^{n} \pi_i r_{ij} \|y_i\| \, \|y_j\| = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1 j\neq i}^{n} \pi_i r_{ij} \|y_i\| \, \|y_j\|+\frac{1}{2} \sum_{j=1}^{n} \sum_{i=1 i\neq j}^{n} \pi_j r_{ji} \|y_j\| \, \|y_i\|,$$

the first two terms in the right hand side of Eqn.(AC-4) become

$$- \sum_{i=1}^{n} \pi_i \left(\frac{\sigma_i}{2}-r_{ii}\right) \|y_i\|^2+ \sum_{i=1}^{n} \sum_{j=1\, j\neq i}^{n} \pi_i r_{ij} \|y_i\| \, \|y_j\|$$

$$=-\frac{1}{2}[\|y_1\|\ \|y_2\| \ldots \|y_n\|]\ M \begin{bmatrix} \|y_1\| \\ \|y_2\| \\ \vdots \\ \|y_n\| \end{bmatrix}.$$

where

$$M = \begin{bmatrix} \sigma_1-2r_{11} & -\pi_1 r_{12}-\pi_2 r_{21} & \cdots & -\pi_1 r_{1n}-\pi_n r_{n1} \\ -\pi_2 r_{21}-\pi_1 r_{12} & \sigma_2-2r_{22} & \cdots & -\pi_2 r_{2n}-\pi_n r_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ -\pi_n r_{n1}-\pi_1 r_{1n} & \cdots & \cdots & \sigma_n-2r_{nn} \end{bmatrix}.$$

If matrix M is positive definite, as stated in Theorem 5-1, then

$$-\sum_{i=1}^{n}\pi_i\left(\frac{\sigma_i}{2}-r_{ii}\right)\|y_i\|^2+\sum_{i=1}^{n}\sum_{j=1\ j\neq i}^{n}\pi_i r_{ij}\|y_i\|\,\|y_j\| \le -\frac{1}{2}\sum_{i=1}^{n}\lambda_m\|y_i\|^2,$$

where $\lambda_m = \min \lambda_i(M)$ is the minimum egenvalue of the matrix M, and Eqn.(AC-5) becomes

$$\dot{v}(y,\phi) \le -\frac{1}{2}\sum_{i=1}^{n}\lambda_m\|y_i\|^2+\frac{1}{2}\sum_{i=1}^{n}\pi_i(-\sigma_i\|y_i\|^2-2\beta_i\|\phi_i\|^2-2\beta_i\phi_i^T\bar{\theta}_i+2y_{oi}\|y_i\|), \qquad \text{(AC-6)}$$

where $y_{oi}=\sum_{j=1}^{n} r_{ij}\|y_{dj}\|$. From the first and the fourth term in the brackets in Eqn.(AC-6), it can be seen that:

$$-\sigma_i\|y_i\|^2+2y_{oi}\|y_i\| = -\sigma_i\|y_i\|^2+2y_{oi}\|y_i\| - \frac{1}{\sigma_i}y_{oi}^2+\frac{1}{\sigma_i}y_{oi}^2$$

$$= -\left(\sqrt{\sigma_i}\|y_i\|-\frac{1}{\sqrt{\sigma_i}}y_{oi}\right)^2+\frac{1}{\sigma_i}y_{oi}^2$$

$$\le \frac{1}{\sigma_i}y_{oi}^2.$$

So (AC-6) becomes

$$\dot{v}(y,\phi) \le -\frac{1}{2}\sum_{i=1}^{n}\lambda_m\|y_i\|^2+\frac{1}{2}\sum_{i=1}^{n}\pi_i\left(\frac{1}{\sigma_i}y_{oi}^2-2\beta_i\|\phi_i\|^2-2\beta_i\phi_i^T\bar{\theta}_i\right).$$

By means of adding and subtracting a term $b_o v$, where $b_o>0$ is a positive constant, to the right hand side of Eqn.(AC-6) and noticing that

$$b_o v = b_o \sum_{i=1}^{n} \pi_i \left(\frac{1}{2} y_i^T P_i y_i + \frac{1}{2} \phi_i^T \Gamma_i^{-1} \phi_i\right) \leq b_o \sum_{i=1}^{n} \pi_i \left(\frac{1}{2} p_i \|y_i\|^2 + \frac{1}{2} \gamma_i \|\phi_i\|^2\right)$$

where $p_i = \max \lambda(P_i)$, $\gamma_i = \max \lambda(\Gamma_i^{-1})$ are the maximum eigenvalues of P_i and Γ_i^{-1} respectively, Eqn.(AC-6) can be rewritten as

$$\dot{v}(y,\phi) \leq -b_o v(y,\phi) + b_o \sum_{i=1}^{n} \pi_i \left(\frac{1}{2} p_i \|y_i\|^2 + \frac{1}{2} \gamma_i \|\phi_i\|^2\right) - \frac{1}{2}\sum_{i=1}^{n} \pi_i \beta_i (\|\phi_i\|^2 + 2\phi_i^T \bar{\theta}_i)$$

$$- \frac{1}{2} \sum_{i=1}^{n} \lambda_m \|y_i\|^2 - \frac{1}{2}\sum_{i=1}^{n} \pi_i \beta_i \|\phi_i\|^2 + \frac{1}{2}\sum_{i=1}^{n} \frac{\pi_i}{\sigma_i} y_{oi}^2. \qquad \text{(AC-7)}$$

Adding terms $\frac{1}{2} \sum_{i=1}^{n} \pi_i \beta_{oi} \|\phi_i\|^2$ and $- \frac{1}{2} \sum_{i=1}^{n} \pi_i \beta_{oi} \|\phi_i\|^2$ to the right hand side of Eqn.(AC-7), and denoting $\frac{1}{2}\sum_{i=1}^{n} \frac{\pi_i}{\sigma_i} y_{oi}^2$ by $\bar{K}_o$, then Eqn.(AC-7) becomes

$$\dot{v}(y,\phi) \leq -b_o v(y,\phi) + b_o \sum_{i=1}^{n} \pi_i \left(\frac{1}{2} p_i \|y_i\|^2 + \frac{1}{2} \gamma_i \|\phi_i\|^2\right) - \frac{1}{2}\sum_{i=1}^{n} \pi_i \beta_i (\|\phi_i\|^2 + 2\phi_i^T \bar{\theta}_i)$$

$$- \frac{1}{2} \sum_{i=1}^{n} \lambda_m \|y_i\|^2 - \frac{1}{2}\sum_{i=1}^{n} \pi_i \beta_i \|\phi_i\|^2 + \frac{1}{2} \sum_{i=1}^{n} \pi_i \beta_{oi} \|\phi_i\|^2 - \frac{1}{2} \sum_{i=1}^{n} \pi_i \beta_{oi} \|\phi_i\|^2 + \bar{K}_o$$

$$= -b_o v(y,\phi) - \frac{1}{2} \sum_{i=1}^{n} (\lambda_m - b_o \pi_i p_i) \|y_i\|^2 - \frac{1}{2} \sum_{i=1}^{n} \pi_i (\beta_{oi} - b_o \gamma_i) \|\phi_i\|^2$$

$$- \frac{1}{2}\sum_{i=1}^{n} \pi_i \beta_i (\|\phi_i\|^2 + 2\phi_i^T \bar{\theta}_i) + \frac{1}{2} \sum_{i=1}^{n} \pi_i (\beta_{oi} - \beta_i) \|\phi_i\|^2 + \bar{K}_o. \qquad \text{(AC-8)}$$

Taking

$$b_o = \min \left[\min_i \left(\frac{\lambda_m}{\pi_i p_i}\right), \ \min_i \left(\frac{\beta_{oi}}{\gamma_i}\right) \right], \qquad \text{(AC-9)}$$

Eqn.(AC-8) can be rewritten as

$$\dot{v}(y,\phi) \leq - b_o v(y,\phi) - \frac{1}{2}\sum_{i=1}^{n} \pi_i \beta_i (\|\phi_i\|^2 + 2\phi_i^T \bar{\theta}_i) + \frac{1}{2} \sum_{i=1}^{n} \pi_i (\beta_{oi} - \beta_i) \|\phi_i\|^2 + \bar{K}_o. \qquad \text{(AC-10)}$$

Since $\|\hat{\theta}_i\|=\|\bar{\theta}_i+\phi_i\|\leq\|\phi_i\|+\|\bar{\theta}_i\|$, it follows that $\|\hat{\theta}_i\|^2\leq(\|\phi_i\|+\|\bar{\theta}_i\|)^2$ and $-(\|\phi_i\|+\|\bar{\theta}_i\|)^2\leq-\|\hat{\theta}_i\|^2$; thus the terms inside the brackets of the second term in the right hand side of Eqn.(AC-10) can be written as

$$- (\|\phi_i\|^2+2\ \phi_i^T\bar{\theta}_i) = - (\|\phi_i\|^2+2\phi_i^T\bar{\theta}_i+\|\bar{\theta}_i\|^2)+\|\bar{\theta}_i\|^2$$

$$= - (\|\phi_i\|+\|\bar{\theta}_i\|)^2+\|\bar{\theta}_i\|^2\leq \|\bar{\theta}_i\|^2 - \|\hat{\theta}_i\|^2.$$

Then Eqn.(AC-10) becomes

$$\dot{v}(y,\phi) \leq -b_o v(y,\phi)+ \frac{1}{2}\sum_{i=1}^{n}\pi_i[(\beta_{oi}- \beta_i)\|\phi_i\|^2-\beta_i(\|\hat{\theta}_i\|^2-\|\bar{\theta}_i\|^2)]+\bar{K}_o. \qquad \text{(AC-11)}$$

For term $(\beta_{oi}- \beta_i)\|\phi_i\|^2$ in the right hand side of Eqn.(AC-11), it has

$$(\beta_{oi}- \beta_i)\|\phi_i\|^2\leq(\beta_{oi}- \beta_i)\|\bar{\theta}_i - \hat{\theta}_i\|^2$$

$$\leq(\beta_{oi}- \beta_i)(\|\bar{\theta}_i\|+\|\hat{\theta}_i\|)^2. \qquad \text{(AC-12)}$$

In order to examine Eqn.(AC-12), it is necessary to recall Eqn.(5-4-18b) which defines the relationships of these parameters in two cases:

(i). if $\|\hat{\theta}_i\|>\bar{\theta}_{oi}$, $\beta_{oi}= \beta_i$; and (ii) if $\|\hat{\theta}_i\|<\bar{\theta}_{oi}$, $\beta_i=0$.

It is easy to see that in case (i), the right hand side of Eqn.(AC-12) becomes zero and in case (ii), Eqn.(AC-12) becomes

$$(\beta_{oi}- \beta_i)\|\phi_i\|^2\leq\beta_{oi}(\|\bar{\theta}_i\|+\hat{\theta}_{oi})^2, \qquad \text{(AC-13)}$$

and will be always true for either case.

Similarly, in case (i), term $-\beta_i(\|\hat{\theta}_i\|^2-\|\bar{\theta}_i\|^2)$ in the right hand side of Eqn.(AC-12) becomes

$$-\beta_i(\|\hat{\theta}_i\|^2-\|\bar{\theta}_i\|^2)\leq\beta_i\|\bar{\theta}_i\|^2\leq\beta_{oi}\|\bar{\theta}_i\|^2, \qquad \text{(AC-14)}$$

and it is zero in case (ii) which means Eqn.(AC-14) holds for both cases.

Thus, in view of Eqns.(AC-13) and (AC-14), Eqn.(AC-11) becomes

$$\dot{v}(y,\phi) \le -b_o v(y,\phi) + \frac{1}{2}\sum_{i=1}^{n}\pi_i[\beta_{oi}(\|\theta_i\|+\hat{\theta}_{oi})^2+\beta_{oi}\|\theta_i\|^2]+\bar{K}_o, \qquad \text{(AC-15)}$$

or

$$\dot{v}(y,\phi) \le -b_o v(y,\phi)+\bar{K}, \qquad \text{(AC-16)}$$

where $\bar{K}=\frac{1}{2}\sum_{i=1}^{n}\pi_i\left[\beta_{oi}(\|\theta_i\|+\hat{\theta}_{oi})^2+\beta_{oi}\|\theta_i\|^2+\frac{1}{\sigma_i}y_{oi}^2\right] >0$, is a constant.

Eqn.(AC-16) is a first order differential inequality, and its solution satisfies

$$v(y(t),\phi(t)) \le e^{-b_o t}\, v(y(0),\phi(0)) + \int_0^t e^{-b_o(t-\tau)}\,\bar{K} d\tau$$

$$= e^{-b_o t}\, v(y(0),\phi(0)) + \bar{K}\int_0^t e^{-b_o(t-\tau)} d\tau$$

$$= e^{-b_o t}\, v(y(0),\phi(0)) + \frac{1}{b_o}(1-e^{-b_o t})\,\bar{K}$$

$$= e^{-b_o t}\,[v(y(0),\phi(0)) - \frac{1}{b_o}\bar{K}] + \frac{1}{b_o}\bar{K}. \qquad \text{(AC-17)}$$

Eqn.(AC-17) states that $v(y(t),\phi(t))$ will converge to a region which is bounded by $\frac{1}{b_o}\bar{K}$ from its initial state $v(y(0),\phi(0))$ with a rate faster than $e^{-b_o t}$, that is,

$$v(y(t),\phi(t)) \le \frac{1}{b_o}\bar{K}, \qquad \text{as } t \to \infty . \qquad \text{(AC-18)}$$

Moreover, in view of Eqn.(AC-1):

$$\frac{1}{2}[\min_i(\pi_i, \min\lambda(P_i))\sum_{i=1}^{n}\|y_i\|^2 + \min_i(\pi_i, \min\lambda(\Gamma_i^{-1}))\sum_{i=1}^{n}\|\phi_i\|^2]$$

$$= \frac{1}{2}[\min_i(\pi_i, \min\lambda(P_i))\|y\|^2 + \min_i(\pi_i, \min\lambda(\Gamma_i^{-1}))\|\phi\|^2]$$

$$\le v(y(t),\phi(t))$$

$$\le \frac{1}{b_o}\bar{K}. \qquad \text{as } t \to \infty.$$

Dividing both sides by

$$\zeta_o = \frac{1}{2} \min \{ \min_i(\pi_i, \min \lambda(P_i)), \ \min_i(\pi_i, \min \lambda(\Gamma_i^{-1})) \} \qquad \text{(AC-19)}$$

$$\|y\|^2 + \|\phi\|^2 \leq \frac{2}{b_o \zeta_o} \bar{K},$$

which is Eqn.(5-4-22) and the theorem is proved.

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